DOCUMENT RESUME

ED 251 324 SE 045 278

AUTHOR	Clarkson, Philip C., Ed.
TITLE	Research in Mathematics Education in Papua New Guinea: 1984. Proceedings of the Mathematics Education Conference, Mathematics Education Centre (4th, Lae, Papua New Guinea. May 1984).
INSTITUTION	Papua New Guinea Univ. of Technology, Lae. Mathematics Education Centre.
REPORT NO	ISBN-9980-76-000-1
PUB DATE	May 84
NOTE	362p.
PUB TYPE	Collected Works - Conference Proceedings (021) -- Reports - Research/Technical (143)
EDRS PRICE	MF01/PC15 Plus Postage.
DESCRIPTORS	*College Mathematics; Concept Formation; *Dropouts; Educational Research; Mathematics Curriculum; *Mathematics Education; *Mathematics Instruction; Mathematics Teachers; Microcomputers; *Problem Solving; Remedial Instruction; Teacher Education
IDENTIFIERS	*Mathematics Education Research; Papua New Guinea

ABSTRACT

Seventeen research reports are arranged under several broad topics of mathematics education. Section 1 concerns language and mathematics learning, with one paper on word problems and one on concept formation. Papers on university level mathematics in Section 2 explore problem solving, characteristics of school leavers, some mathematical aspects of surveying education, an experimental approach to teaching numerical analysis, and remediation. In the third section, reports consider mathematical deficits of university entry students and topics to consider in the curriculum. The fourth section concerns post year 10 mathematics, with reports on the quality of school leavers and on distance versus face-to-face learning. Mathematics teaching in various cultures is the focus of Section 5, with secondary mathematics teachers, mathematical abilities, basic skills, and secondary teacher education in Uganda considered. Finally, problem solving with a function graphing program and a report on microcomputer use in Fiji are presented in Section 6. (MNS)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED251324

RESEARCH IN MATHEMATICS EDUCATION IN PAPUA NEW GUINEA

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)
✓ This document has been reproduced as received from the person or organization originating it.
Minor changes have been made to improve reproduction quality.

- Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

1984

P.C. CLARKSON
(ed.)

SE 045 278

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Phil Clarkson

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

RESEARCH IN MATHEMATICS EDUCATION

IN

PAPUA NEW GUINEA:

1984

EDITED BY

PHILIP C. CLARKSON

Proceedings of the Fourth
Mathematics Education Conference,
Mathematics Education Centre,
Lae,
May, 1984.

Published by:

Mathematics Education Centre
Papua New Guinea University of Technology
Lae
Papua New Guinea
1984
ISBN: 9980 - 76 - 000 - 1

CONTENTS

SECTION 1 - Language and Mathematics Learning

Word Problems at Year 5 Level 2

P.C. Clarkson - Mathematics Education Centre, Papua New Guinea University of Technology

First and Last, One and Two, Letter and Word: Concept Formation in Papua New Guinea Community Schools 19

D. Moore - Education Research Unit, University of Papua New Guinea

SECTION 2 - University Level Mathematics

What Problems Can Tertiary Students Solve? 33

C. Wilkins - Mathematics Department, Papua New Guinea University of Technology

A Mathematics Profile of Grade 10 Leavers at Papua New Guinea University of Technology, 1984 45

H. Whippy - Mathematics Department, Papua New Guinea University of Technology

Some Current Mathematical Aspects in Surveying Education 59

P. Done - Surveying Department, Papua New Guinea University of Technology

An Experimental Approach to Teaching Mathematics 133

[illegible]. Joyce - Mathematics Department, University of Papua New Guinea

Minimal Remediation: Pre-test as a Kick in the Pants 147

J. Gough - Education Department, University of Papua New Guinea

SECTION 3 - Viewing School Maths from a University Perspective

Are Grade 12 University Entry Students Deficient in Their Mathematics Education? 157

K. Collard - Mathematics Department, Papua New Guinea University of Technology

Is There a Place for this Kind of Thing.....? 177

B. Landau - Mathematics Department, Papua New Guinea University of Technology

SECTION 4 - Post Year 10 Mathematics

The Quality of the Grade 10 School Leavers Pool 189

L. Ros - Mathematics Department, Goroka Teachers College (University of Papua New Guinea)

Distance Versus Face-to-Face Learning: A Mathematics Test Case 199

G.S. Kaeley - Extension Studies Department, University of Papua New Guinea

SECTION 5 - Mathematics Teaching in Various Cultures

What is it Like Being a Mathematics Teacher in a Secondary School in Papua New Guinea? 231

N. Wilkins - Mathematics Education Centre, Papua New Guinea University of Technology

Mathematical Abilities at the Upper Secondary School in Fiji 243

L. Raj - School of Education, University of South Pacific, Fiji

Basic Computation Skills Progress in Grade 10 Consumer Mathematics 253

D. Nelson - Ministry of Education, Yap, Federated States of Micronesia

An Innovation in Secondary Mathematics Teacher Education in Uganda 261

M. Magino - Ministry of Education, Uganda

SECTION 6 - Computers and Mathematics Teaching

Some Problem Solving with a Function Graphing Program - A Workshop 273

R. Seldon - Mathematics Department, Papua New Guinea University of Technology

Microcomputers in Education in Fiji 289

F. Dewa - School of Education, University of South Pacific, Fiji

Conference Program

1. LANGUAGE AND MATHEMATICS LEARNING

GRADE 5 PAPUA NEW GUINEAN STUDENTS' PERFORMANCE ON MATHEMATICAL WORD PROBLEMS

PHILIP CLARKSON

In a recent review of studies which investigated the link between mathematical learning and language learning in Papua New Guinea, it was concluded that:

1. There is a concern at all levels of the education system with the quality of mathematical learning and a suspicion that language difficulties are partly the cause of problems in mathematics learning.
2. There is evidence to show that students and probably a number of primary school teachers have difficulty with many important mathematical words and the underlying concepts.
3. There is some evidence which indicates that students do not analyse questions carefully.
4. There is a suggestion that the type of relationship between mathematical learning and language learning may well change as students progress through the system.

(Clarkson, 1984).

In particular it was noted that during the middle years, grades 4 to 10, students are weak at solving word problems.

The review also suggested that all the studies focussed on competency in English and related this to mathematical learning. No study known to the author had ascertained whether competency in their mother tongue and/or 'Pidgin English' has an effect on mathematical learning.

It was felt within the Mathematics Education Centre (M.E.C.) that there was a need to confirm the suspicions raised by past studies and explore the relationship between mathematics and language in a detailed project. Thus the present project titled 'Language and Mathematics' began.

It was decided to work with grade 6 students to begin with. This is a vital year in the education system within Papua New Guinea since on average only 60% of the year 6 population will progress to secondary school. As well previous studies had suggested that by year 6 language factors were having a decided influence on a student's mathematical performance.

A previous study (Clarkson, 1983), had used the Newman technique of error analysis to categorize the types of errors a group of grade 6 students made on mathematical problems. Among the results it was shown that 12% of the errors were of a reading type, and 21% were of a comprehension type. It was decided in the present study to again use the Newman technique and relate the frequency of reading and comprehension errors to such variables as:

- language competency in written and verbal English.
- language competency in written and verbal Pidgin.
- competence on a test of logical connectives in English.
- measure of cognitive levels.
- measures of cognitive styles.
- measure of attitude to mathematics.

The first step in the project was to devise a test composed of mathematical word problems suitable for Papua New Guinean students. The errors students made on this test would be investigated using the Newman technique. This paper describes the trialling of this test and analyses some of the resulting data.

THE MATHEMATICAL WORD TEST

Two national senior research officers of the Mathematics Education Centre began the process by composing a large number of problems on the topic areas of number and measurement. They used the Community School grade 5 syllabus as a guide to the content and difficulty of the questions. These questions were reviewed a number of times by staff at the Mathematics Education Centre and by a number of Community School teachers. This review process left a bank of 96 questions.

The 96 questions were split randomly into four tests of 24 questions each. The four tests were then administered to 512 grade 5 students in seven different Community Schools in the Lae area at the end of 1983. It was felt that the grade 5 students could be considered as grade 6 beginners. Approximately twelve copies of each test were taken into a class and these were distributed randomly to the students. A time limit of 50 minutes was allocated for the administration of the test. There were very few students who were not quite satisfied with this amount of time.

Each question was scored either right or wrong and allocated a mark of one or zero respectively. Percentage difficulties and discrimination coefficients were calculated for each item. On the basis of these statistics, twenty questions were selected so that there were four items on each of the four operations, three items on fractions and one item on number. This then was the resultant test to be used in the main study.

RELATIONSHIP BETWEEN WORD PROBLEMS AND NUMBER PROBLEMS

It occurred to the author that while the four trial tests of word problems were being tried out, it would be an opportunity to investigate the relationship between a student's ability at solving word problem and pure number problems. Thus items were composed which reflected the most likely number sentence which would be used by a student in solving a particular word problem in the opinion of the Mathematics Education Centre staff. Thus,

> What is the sum of twenty and fifty two?
> was mirrored by;
> 20 + 52 =

Of course one cannot be completely sure that the number sentence would in fact be what the students used. A post interview would need to be held to be certain. However based on the experience of the staff of the Mathematics Education Centre, we were reasonably confident that most number sentences would mirror what the students did.

Hence a second test of 24 appropriate number problems was administered to each student immediately after the test of word problems. Thirty minutes was allowed for this test which again seemed quite adequate from the reaction of the students.

In composing the number problems there were occasionally word problems which did not have a direct number sentence equivalent; for example reading the time off a pictured clock face. A substitute simple number sentence was composed for such a question so that each test of number type problems was still composed of 24 items. In the subsequent analysis of the results, when it seemed important to mirror the word problem with its strict number type problem, these substitute questions have been dropped from the analysis.

STATISTICS OF THE TESTS USED

Table 1 shows the means, standard deviations and correlations of the eight tests; the four word type and the four number type.

Table 1

Statistics from eight tests

(Possible top score on each test was 24)

	N	WORD $\bar{X}$	TYPE S.D.	NO. $\bar{X}$	TYPE S.D.	CORR.
TEST A	122	8.0	4.0	11.5	3.3	0.61
B	128	7.8	4.6	12.2	4.5	0.69
C	134	5.5	3.5	9.4	4.6	0.73
D	128	7.8	4.4	10.7	4.4	0.71

The averages from each type of test are low. Sullivan's (1981) comments, confirmed later by Southwell (1984), that year 10 students can handle pure number problems but have difficulty with word problems do not apparently apply to grade 5 students. They seem to have difficulty with both types of problems.

BEST COPY AVAILABLE

The correlations between the tests are reasonably high suggesting some sort of relationship of the type that the students who perform well on the number type problems also perform reasonably well on the word type problems.

ITEM ANALYSIS

Percentage difficulties for the items were calculated. A comparison between the item difficulties of corresponding number and word type items was made. Ten questions were dropped out of this comparison since the word types did not have corresponding number sentences for their solutions, for example, reading the time on the clockface. This left 86 questions in the analysis. Table 2 shows this data.

Table 2

Average percentage difficulties of the two tests and their correlation

Word Type	$\bar{X}$	68.8
	S.D.	32.2
No. Type	$\bar{X}$	54.1
	S.D.	32.0
Correlation		0.69

It appears from Table 2 that on average two thirds of the students obtained a wrong answer for each word type problem. In comparison a little over a half of the students got each number problem wrong. The value of the correlation coefficient suggests that if students were getting an item of one type wrong, they were getting the corresponding item wrong as well.

Table 3

Comparison of item difficulties between corresponding items.

Percentage of word items more difficult than the number items	70%
Percentage of word items with 5% greater difficulty than corresponding number items	57%
Percentage of number items with 5% greater difficulty than corresponding word items	15%

The comparison of the item difficulties between corresponding questions shown is Table 3 does support the above suggestions that in general the word type problems are more difficult. However, this is not true for all item pairs. For 13 of the 86 pairs (15%) the word problem were easier. These items were examined more closely.

- 7 of the 13 were story type questions compared with a straight instructional type like 'What is the remainder when 29 is divided by 6?'
- Of the 13, 1 was addition
  4 were subtraction
  2 were multiplication
  4 were division
  2 were fractions
- 5 were measurement type problems.
- There was a mixture of problems requiring carrying and non-carrying.
- There was a mixture of multi-digit numbers and single digit numbers.

There seemed to be no obvious common factor.

COMPARISON OF CONTENT

The questions were separated into categories of whether they were essentially solved by one of the four arithmetic operations or were predominantly examining length or time concepts. Table 4 shows this breakdown with the average percentage difficulty of each group.

Table 4

Analysis of questions by content.

(The number in brackets in the category column indicates the number of questions for that category. Other numbers in brackets are standard deviations).

CATEGORY	WORD TYPE AVER. % DIFF.	NUMBER TYPE AVER. % DIFF.	CORRELATION
Addition	56.3	28.0	.38
(10)	(25.4)	(23.2)	
Subtraction	60.9	47.7	.75
(18)	(23.5)	(31.8)	
Multiplication	61.2	38.4	.69
(18)	(20.0)	(31.6)	
Division	66.3	67.3	.91
(15)	(22.0)	(29.0)	
Length	88.1		
(10)	(8.3)		
Time	86.8		
(9)	(8.5)		

An examination of Table 4 indicates that for the word type problems all the categories were difficult. The order of easiest group to hardest was addition, subtraction and multiplication about the same, division, and then a big jump to time and length. This order for the operations is to be expected. It could be speculated that the big jump in difficulty to the measurement topics was because of the presence of technical terms and symbols such as *m*, *cm*, etcetera.

The number type questions which paired the measurement topics need to be left to one side since they were a mixture of the four oeprations. Of the other four categories, the order of easiest to hardest was addition, multiplication, subtraction and division. Interestingly this was the same order that Hart (1931) found in her work in England; that is multiplication is easier than subtraction.

An examination of the correlation coefficients gives an indication of a strong relationship between the two types of questions for the subtraction, multiplication and division operations, but a weaker relationship for addition. The strength of relationship as measured by the correlation coefficient follows the same order as the percentage difficulty. It may be that as students overlearn their number facts (as may have happened with the addition facts by grade 5), the relationship between the ability in solving number problems and a corresponding word problem weakens. If this hypothesis was shown to be correct, it still leaves the word problems relatively difficult. Perhaps another indication of the role of language in this whole process.

SPECIFIC WORDS

In talking about an arithmetic operation teachers use different words to imply the use of that operation. For example when working with addition the following may be used:

add, sum of, added to, plus, more than

and so on. If students are to understand fully the concept of addition they should be familiar with all the different terms used. Obviously the same goes for the other three operations.

Items which were of a non story type but a straight instructional type were isolated. For example:

Add five and eleven.

From this group of items, their corresponding number type questions were examined. Any item for which the percentage difficulty of the number type was greater than 20 were discarded. This was an attempt to control for the effect of number difficulty. This resulted in thirteen word type problems. These were grouped by the operation that was used to solve the problem. Table 5 shows the items by the particular term that embodies the operation.

Table 5

A comparison of the percentage difficulty of problems which used different words for the various operations

ADDITION	% DIFFICULTY
Add	16
Sum of	42
Sum of	70
More than	85
SUBTRACTION	
Subtract	32
Minus	40
Minus	41
Subtracted	42
Less than	53
Difference	71
MULTIPLICATION	
Three fives	39
Multiply	43
Product	57

It will be noted that there are no division types. All the corresponding number items had percentage difficulties greater than 20.

This data is not conclusive by any means. However it is interesting to note the range of percentage difficulties for each operation. This does raise the suspicion that students (and perhaps teachers as well) maybe keying into one or maybe two terms and use them exclusively. This is a dangerous practice. When these students reach Provincial High School, the number of teachers will be much greater and will probably use a range of terms. Perhaps more importantly, the material these students hopefully read outside the school room will use a variety of terms. Students should be introduced to them while they are at school.

TEACHERS' ASSESSMENT OF QUESTIONS

Teachers were asked to rate the written type questions on two five point scales. The first consisted of a five point scale which questioned whether the items were satisfactory or not for beginning grade 6 students. That is, would the teacher be prepared to use the question in his/her class. The five point scale had headings of very satisfactory, satisfactory, alright, unsatisfactory, very unsatisfactory.

The second scale investigated whether the teachers felt each question was difficult or easy. The five point scale had headings of very difficult, difficult, alright, easy and very easy. They again were asked to consider the questions in relation to an average student just beginning grade 6. In addition, if they rated a question as difficult, the teachers were asked to indicate why it would be so.

The teachers who responded to this instrument consisted of two groups. The first attended a workshop in Lae to consider mathematics teaching. These teachers worked in pairs on different sets of 24 problems selected at random from the 96. The second group of teachers were teaching in Community Schools in the Lae area and had not attended the workshop. The full set of 96 questions were sent to their schools and the grade 6 teachers were asked to respond.

Some teachers misunderstood the directions although these had been checked with a small group of national teachers before hand. Often only one of the scales was used to rate a question. Hence there was not a high rate of response for the second scale.

Analysing the response to the first scale which dealt with whether the questions were satisfactory for beginning grade 6 students it was found that at least ten scripts referenced each question for this scale. One suspects this represented the views of 14 or so teachers since it will be remembered that in the workshop situation the teachers worked in pairs. There were only two questions which were noted as unsatisfactory. This classification was made on only one script. It appears from the comments made that the teacher(s) did not know how to work out the answer. Hence there is some assurance that in the estimation of the teachers these questions were satisfactory. This is important for the major project to follow. There is some justification in believing that these questions may well represent the type which teachers would use in their classrooms.

The second scale dealt with the difficulty of the questions. As noted above this scale was sometimes not used. It was decided to only analyse questions for which at least five scripts commented. This would represent the views of 8 to 9 teachers. Four items were dropped out leaving 92.

The results were analysed using a five point scale: very difficult was scored 1 with very easy scored 5. Table 6 shows the results with the students measured facility.

Table 6

Teachers' rating of the questions on the difficulty scale. Students' measured facility is also shown with the correlation between the two measures.

	$\bar{X}$	S.D.
Teachers' Rating	3.6	0.7
Students' Measured Facility	31.0	22.1
Correlation	0.46	

Analysing the data in Table 6 it can be seen that the teachers believed the questions are a little on the easy side of 'alright'. On the other hand the measured student facility shows that on average the students found the questions difficult. The correlation coefficient as far as educational considerations are concerned is on the low side. This suggests that teachers are not judging the difficulty of new questions very well. It would be interesting to compare this finding with data collected in other countries to see whether this is a general problem with teachers.

Very few teachers gave reasons as to why they felt a question would be difficult. Those that did often noted:

- what the answer should be, or
- the way to get the answer,

neither of which are really reasons.

A few teachers:

- noted that students will find a type of arithmetic operation difficult, or fractions are difficult, or conversion between units are difficult.

There was only one comment that referred to a difficulty with the understanding of a specific term. There were no comments which referred to the general language context as being a cause of difficulty.

This data suggests then that as well as teachers not judging the difficulty of questions very accurately, they also have a problem with spotting where the difficulty will be. Certainly they do not consider language problems as an important factor. This data is suggestive only. A far wider sample of teachers is needed for definitive conclusions and recommendations to be made. However there is some evidence to support the idea that this is an area of concern. It was interesting to note that at the 1980 and 1981 annual PNG National Mathematics Teachers' Workshops, there were sessions on how to write good questions.

SUMMARY

The following points can be made from the foregoing analysis. They may suggest lines of future investigation.

1. Students finishing grade 5 find word problems very difficult.

2. Students finishing grade 5 do not have mastery of number operations.

3. There is a dependence of word problems on the underlying arithmetic operation, but a suggestion that even when the arithmetic operation is mastered there is still a large residual difficulty with word problems.

4. There is a big difference in difficulty between word problems solved by applying one of the four arithmetic operations and those dealing with length or time. Is it the technical terms which cause this?

5. For the number type problems multiplication was found to be easier than subtraction.

6. There was an indication that students (and teachers) may use only one specific word to talk about an arithmetic operation.

7. There was an indication that teachers do not judge the difficulty of word questions very well. They probably need help in this area.

8. There was an indication that teachers don't recognize that the difficulty of word problems is partially caused by language factors.

REFERENCES

CLARKSON, P.C. 'Language and Mathematics in Papua New Guinea: A Land of 720 Languages', in *Proceedings of the Eighth International Conference for the Psychology of Mathematics Education* ed. by B. Southwell, R. Eyland, M. Cooper, J. Conroy and K. Collis, Sydney, International Group for the Psychology of Mathematics Education, 1982, pp. 331-340.

CLARKSON, P.C. 'Types of Errors Made by Papua New Guinean Students' *Educational Studies in Mathematics* 14, 355-367, 1983.

HART, K.M. (ed) *Children's Understanding of Mathematics: 11-16* London, John Murray, 1981.

SOUTHWELL, B. 'Problem Solving With Calculators for Grade 10 Students', *Mathematics Education Centre Technical Report No. 16* Papua New Guinea University of Technology, 1984.

SULLIVAN, P. 'The Standards Debate: An Alternate Strategy' *Mathematics Education Centre Report No. 15* Papua New Guinea University of Technology, 1981.

FIRST AND LAST, ONE AND TWO, LETTER AND WORD : CONCEPT FORMATION IN PAPUA NEW GUINEAN COMMUNITY SCHOOLS.

D W MOORE

Educational Research Unit

Paper presented to the 4th National Conference on Mathematics Education, Lae, Friday May 25, 1981.

ABSTRACT

In recent years emphasis has been placed on the relationship between pupil language competencies and the learning of mathematics in Papua New Guinea. Jones (1981,a) for example suggesting that improvement in the quality of mathematics learning in Community Schools was unlikely without prior emphasis on improving pupils' reading skills. Similarly reading researchers (e.g. Clay, 1972) have been attending to the importance of a number of specific, testable, pre-reading concepts - the acquisition of which is crucial to the comprehension of written language. This paper presents data illustrating the rate of acquisition of a set of these concepts in Community Schools and considers some implications of this learning pattern for both mathematics and reading instruction.

First and Last, One and Two, Letter and Word : Concept Formation in Papua New Guinean Community Schools.

In recent years a number of mathematics education specialists have focussed on the relationship between language and mathematics education (Wilson, 1981). In Papua New Guinea several researchers have emphasized the importance of the pupil's (English) language competence as a factor in the teaching/learning of mathematics in Community Schools (Allen, Thomas, & Patu, 1975; Clarkson, 1984; Lancy, 1983; Souviney, 1981). Jones (1981,a) presented data indicating that limited proficiency in reading can restrict a child's ability to determine the mathematical relationships that underlie certain types of word problems, and suggested that an emphasis an improving **reading** skills in Community School pupils might be a precondition to any significant gains in their mathematics learning. Meek and Feril (1978), testing comprehension of mathematics vocabulary, reported that grade 6 students could comprehend only about 20% of the mathematics related words in grade 5 mathematics lessons and, perhaps more damagingly, that a sample of teachers-in-training could also understand only about half of this vocabulary.

In a similar vein Jones (1981,b) traced the functional acquisition of the relational concepts "more" and "less" in comparative, direct, and indirect contexts in pupils attending Community and International Schools in Papua New Guinea. He reported that whereas mastery of these concepts in all three context was achieved by grade 6 in the International School sample (all of whom were native English speakers) a quarter of the grade 10 pupils who had come through the Community School system were still having difficulty with "more" and "less" in the indirect context. Jones noted that such delay in the formation of basic number concepts could exacerbate the mismatch between the language proficiency of the pupils and, in particular would limit their ability to solve arithmetical word problems. He concluded "...if serious progress is to be made in improving the quality of mathematics instruction for those learning mathematics in a second language, more needs to be known about those aspects of language which impinge directly on the learning of mathematics and which present particular difficulties for those learning in their second language" (p. 21).

Simultaneously, a number of researchers (see for example Clay, 1972,a; Downing & Oliver, 1974; Ried, 1966) concerned with the early development of reading skill have focussed on orthographic knowledge development and also on the child's comprehension of the language of instruction - language described by Downing (1976) as the reading instruction register. These researchers have found that many children who are beginning reading instruction do not understand the reading instruction register - and that childrens' knowledge of this, together with their awareness of the conventions of print, are important bases for the successful teaching of reading. As Day, Day, Spicola, & Griffen (1981, p 77) observed, "the child who does not understand concepts such as "word", "sentence," "first letter," etc., that are commonly used by teachers during reading instruction ...may... receive less than maximum benefit from such instruction."

Arising directly from this research school, and focussing originally on those concepts first required by the reading learner, Marie Clay (1972, b) developed the **Concepts About Prints** test. This instrument was designed to be administered by teachers as part of a diagnostic survey of their pupils' reading skills after one year of schooling, and to assist in the early detection (and remediation) of reading difficulties.

The three main objectives of the **Concepts About Print** test are:

1. The precise observation of what a child is doing while reading
2. Uncovering the initial reading processes a child controls
3. Discovering reading behaviours which need to be taught.

The test uses a 20-page children's book entitled **Stones** which the pupil is asked to help the interviewer read. During the 10-minute story/interview the child responds to 24 questions concerning print directionality, capitalization, basic punctuation, concepts about the relationship between written and oral language, as well as several relational concepts and the words "letter" and "word."

Norms provided with the test indicate that the average child **in New Zealand schools** will have mastered all concepts measured by this test within the first two years of schooling. Clay emphasized however that these norms are most curriculum dependent. Differences in both the rate and sequence of concept acquisition should be expected as a function of difference in teaching program or method emphasis.

This test has since been extensively trialed in a variety of school and language systems (Spanish and Danish translations are now available) and has been shown to have acceptable reliability and validity (Day & Day, 1980; Hollingsworth, 1978). Johns (1980), testing a sample of American children at the end of grade 1, found significant differences between above-average and below-average readers in print-direction, letter-word, and advanced-print concepts. Day, et.al. (1981), in a longitudinal study of the development of orthographic awareness (i.e. knowledge of print) in children during kindergarten and first grade, found a strong relationship between the level of such awareness at the beginning of first grade and pupils' end of year reading attainments, thereby illustrating the important part a firm understanding of these initial concepts plays on subsequent reading skill acquisition.

Schmidt and Jansen (1981) reported a comparative study of the development of these concepts in United States and Danish children. Whereas New Zealand children (Clay's original test group) enter school at age five years, school begins at age six and seven respectively in the USA and Denmark. This study therefore provides a valuable comparison of the rate of acquisition of these concepts about print in quite different school systems. Although several interesting differences were noted,- in particular the American children showed a more rapid development of orthographic awareness upon initial entry to school - children in both countries had mastered most of the skills tested after one, or certainly two years at school. Indeed by the beginning of Danish grade 2 there was no significant differences between these children's results and those of their American age peers - then beginning grade 3. Thus it appears that at least by the end of grade 2 in both these countries children can be expected to have acquired a quite robust idea of how a book should be oriented for reading, the conventions of print directionality, important metalinguistic concepts such as "letter" and "word", "first" and "last", "one" and "two", some basic punctuation, and a more rudimentary understanding of more advanced print concepts such as those involving word recognition and the visual discrimination of fine print detail; in short, a solid foundation for the subsequent development of more advanced reading skills.

Four questions in this test are designed specifically to assess the pupil's comprehension of the concepts "first" and "last", "one" and "two", and "letter" and "word", the first four of which are basic mathematical concepts which are also very much a part of the reading instruction register. The purpose of this paper is to document the development of these concepts in a sample of rural Community School children[1].

Method

The participants were 75 girls and 75 boys enrolled in one of three, six-class, rural Community Schools in the Eastern Highlands and Morobe Provinces. Five girls and five boys selected (by random numbers from the class register) from grade 2 through grade 6 inclusive in each school were individually assessed on the **Concepts About Print** test.

Goodman's (1981) review of this test queried the cross-cultural suitability of the children's story book and especially the illustrations provided. With this in consideration the test booklet was adapted for use in Papua New Guinea. Pictures were redrawn to depict a Papua New Guinean child in a more readily identifiable cultural setting. The original text and layout was retained with one exception. The noun "swing" was once replaced by the noun "fence" again with the intention of increasing the story's suitability to rural Papua New Guinean children. All testing was carried out during weeks 8 and 9 of the school year.

---

[1] For a full report of the pupils' performance of the **Concepts About Print** test see Moore (1984).

An analysis of variance was performed on all items of the Concepts About Print test according to grade and sex. Post hoc pairwise T tests, adjusted for multiple comparison by the Bonferroni formula (Dixon, 1983) were performed on those items showing significant differences to identify where these lay.

Results

Sex of subject had no significant main or interactive effects on responses to these questions. Consequently the results are collapsed across sex for the remaining discussion.

First and Last

Two items from the Concepts About Print test sample the respondents comprehension of these concepts:

item 7. "Show me the first/last part of the story."

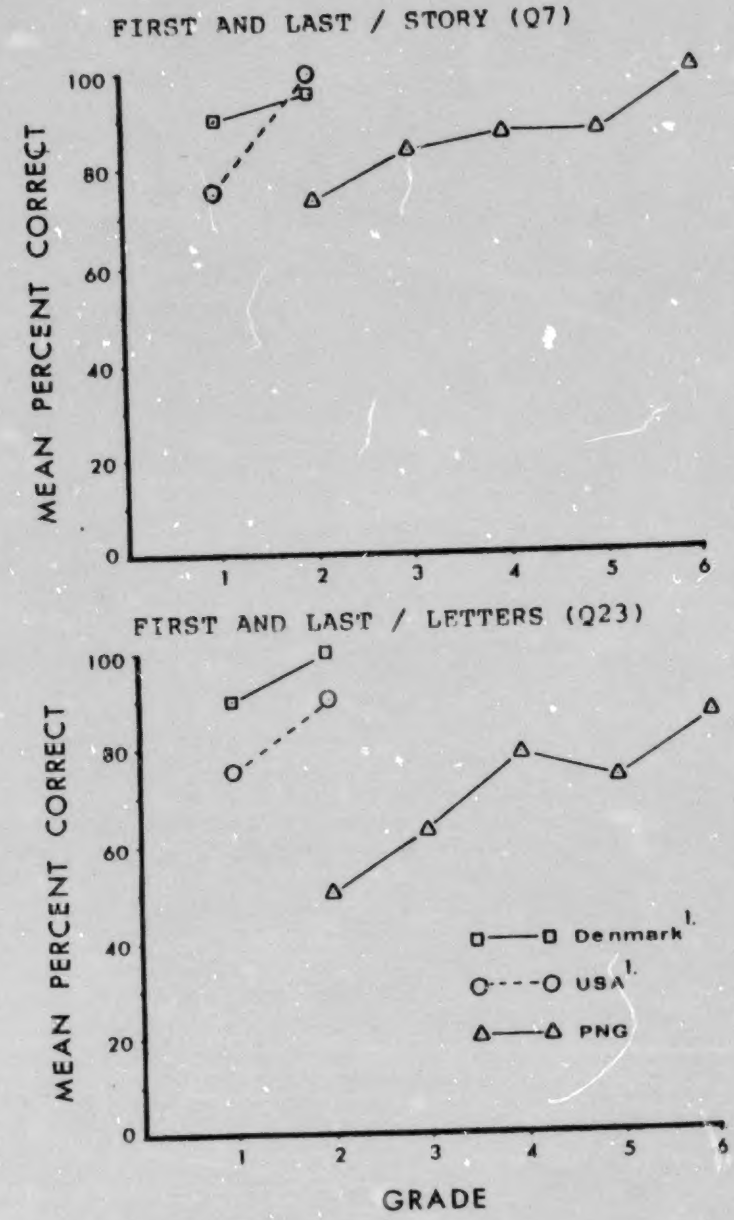
item 23. "Show me the first/last letter of a word."

Figure 1 shows the percent of pupils at each grade level answering these test items correctly. Grade equivalent scores from samples of United States and Danish children (Schmidt & Jansen, 1981) are also presented for comparative purposes.

As is illustrated in these graphs a higher proportion of Community School children at each grade level could functionally demonstrate understanding of these concepts in the more general (story) context than when applied to letters in a word. However only 73% of the grade 2 pupils passed the first item and not until grade 6 did all pupils in the sample correctly identify the first and last parts of the story (ANOVA F = 2.23, df = 4, p >.05). In the more difficult context (item 23) 50% of grade 2 pupils passed, this proportion increasing by grade 6 to 86%. (ANOVA F = 3.16, df = 4,p<.05). Post hoc pairwise T tests revealed the only significant difference between pairs to be between grade 2 and 6 (T =-3.34, df 52, p<.005) suggesting that Community School pupils continue to acquire these concepts throughout the six years of enrollment.

One and Two, Letter and Word.

Comprehension of these concepts are also tapped by two items in



1. from Schmidt & Jansen, 1981.

Figure 1. Percentage of pupils at each grade correctly identifying "first" and "last".

the **Concepts About Print** test:

item 21. "...push the cards across the story like this until all you can see is just one letter"

"Now show me two letters" (credit if both are correct)

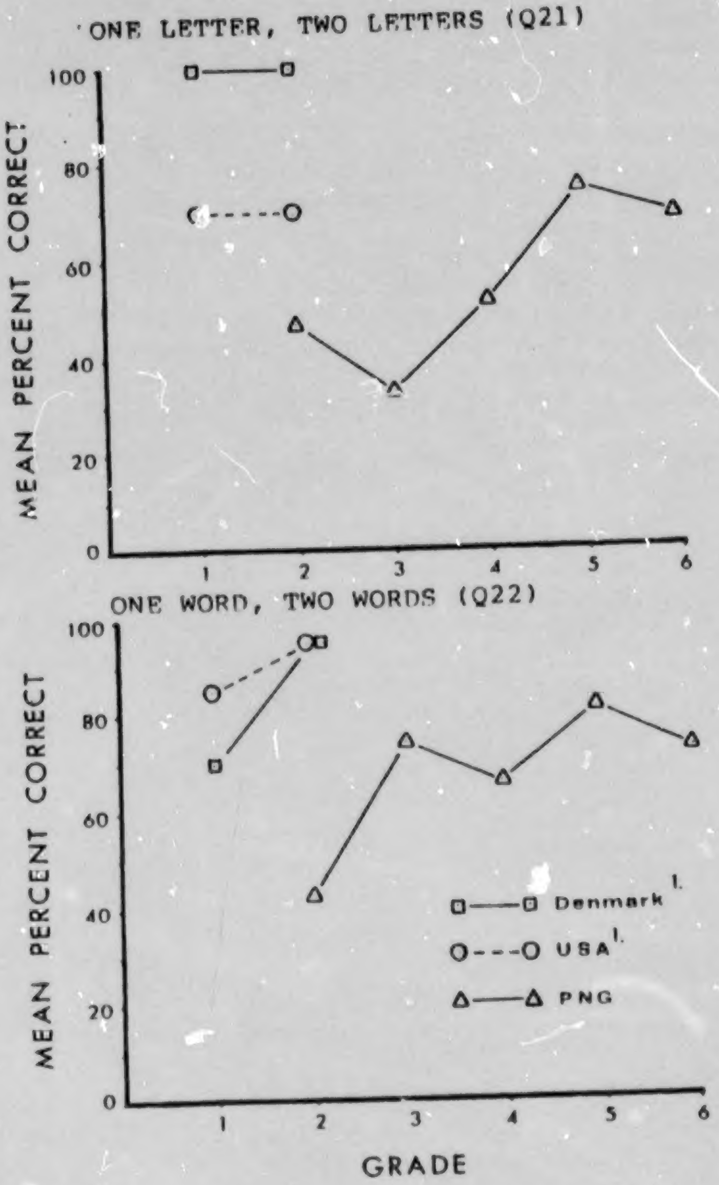
item 22, "Show me just one word", "show me two words" (credit if both are correct)

Figure 2 shows the percentage of pupils passing these items at each grade. Again it appears that the finer context (letter) proved more difficult than the other, especially in the lower grades.

Although both these items are phrased in such a way that it is not possible to determine the relative effects of the seperate concepts of "oneness and twoness" and "letter and word", it is evident that in combination these concepts are causing difficulties to a large proportion of these Community School children at all grade levels, and again that these basic concepts are being continuously developed throughout the enrollment period. (item 21, ANOVA $F = 4.11$, $df = 5$ $p<.005$, post hoc T tests across all pairs show a significant closest pairs difference between grades 3 and 5, $T = -3.62$, $df = 57$, $p<.005$. **Item 22,** ANOVA $F = 3.92$, $df = 4$, $p<.005$, post hoc pairwise T tests reveal the only significant difference to occur between grades 2 and 5, $T =-3.62$, $df = 55$, $p<.005$).

Discussion

The research data from the USA, Denmark and New Zealand show variations in the rate and sequence in which these skills and concepts are learned; variations which reflect differences in school entry age, prior experience, and particular curriculum emphases. However the present results suggest that the rate of acquisition of these concepts in Papua New Guinean Community Schools is of a different order to that observed previously. Whereas the Clay (1972,b) Day et al (1981), and Schmidt and Jansen (1981) data indicate that the average child in their school systems has learned practically all the skills and concepts sampled in the **Concepts About Print** test by the end of the second year at school our pupils are continuing to develop these concepts throughout Community School. Indeed by the beginning of grade 6 a sizeable proportion of these pupils continue to display deficits in their understanding of the concepts "first" and "last","one" and "two", "letter" and "word".



ONE LETTER, TWO LETTERS (Q21)

ONE WORD, TWO WORDS (Q22)

1. from Schmidt & Jansen, 1981.

Figure 2. Percentage of pupils at each grade correctly indicating "one" and "two", "letter" and "word".

Previous recommendations to reading teachers (Clay, 1972,a; Day et al. 1961; Downing, 1976) that they should not assume that all their pupils understand terminology commonly used in reading instruction and other basic concepts appear particularly apposite to mathematics teachers in Papua New Guinea.

It should be noted that children in the USA, Denmark and New Zealand are all raised in largely literate and print-rich environments, and upon entry to school most are taught in their maternal language. Neither is true in Papua New Guinea. Consequently one might anticipate greater initial confusion and a less precocious development of these concepts on the part of Papua New Guinean pupils. For the same reasons it is especially important that the Community Schools provide suitable instruction for these pupils to ensure the acquisition of such basic concepts upon which more advanced skills can develop.

The author wishes to express his thanks to those Headmasters, teachers and especially the pupils who participated in this study and to acknowledge in particular the assistance of Gail Edoni who carried out most of the testing.

Correspondence concerning this paper should be addressed to Dennis Moore, Education Research Unit, Box 320 Uni P.O., University of Papua New Guinea.

References

ALLEN,A.L., THOMAS,N.D., & PATU,P.(1975): Abilities of newly trained indigenous secondary mathematics teachers in Papua New Guinea, **Papua New Guinea Journal of Education**, 11(1), 1-7.

CLARKSON,P.(1984): **Word problems at year 6 level.** Paper presented at the Fourth National Mathematics Education Conference. Lae, Papua New Guinea.

CLAY,M.M.(1972,a): **Reading: The patterning of complex behaviour.** Auckland: Heinemann Educational Books.

CLAY,M.M.(1972,b): **The early detection of reading difficulties: A diagnostic survey.** Auckland: Heinemann Educational Books.

DAY,H.D., & DAY,K.C.(1980): The reliability and validity of the Concepts About Print and Record of Oral Language. **Resources in Education**, ED179932.

DAY,K.C., DAY,H.D., SPICOLA,R., & GRIFFEN,M.(1981): The development of orthographic linguistic awareness in kindergarten children and the relationship of this awareness to later reading achievement. **Reading Psychology**, 2(2), 76-87.

DIXON,W.J.(1983): Description of groups (strata) with histograms and analysis of variance. In DIXON,W.J. (Ed) **B.M.D.P. Statistical software.** Berkeley, University of California Press.

DOWNING,J.(1976): The reading instruction register. **Language Arts**, 53, 762-766.

DOWNING,J., & OLIVER,P.(1974): The child's concept of a "word". **Reading Research Quarterly**, 9(4), 568-582.

GOODMAN,Y.M.(1981): Review of Concepts About Print. **The Reading Teacher**, 34, 4-9.

HOLLINGSWORTH,S.(1978): An investigation of the cognitive, environmental, and personal differences in the reading performance of preschool children. **Dissertation Abstracts International**, 38, 4667-A.

JOHNS,J.L.(1980): First graders' concepts about print. **Reading Research Quarterly**, 15(4), 529.

JONES,P.L.(1981,a): Solving word problems in a second language and reading proficiency. In CLARKSON,P. (Ed.) **Research in mathematics education in Papua New Guinea.** Lae, Mathematics Education Centre Press.

JONES.,P.L.(1981,b): Learning mathematics in a second language. **Mathematics Education Centre Report No 19.** Lae, Mathematics Education Centre Press.

LANCY,D.F.(1983): **Cross-cultural studies in cognition and mathematics.** New York, Academic Press.

MEEK,C., & FERIL,N.(1978): A pilot study into the level of comprehension of some words taken from the New Guinea Community School Mathematics Project. **Tech. Report 3.** Lae, Mathematics Education Centre.

MOORE,D.W.(1984): **Acquiring the initial concepts about print in Papua New Guinea Community Schools.** Paper presented in the UPNG Education Faculty Research Seminar series.

REID,J.F.(1966): Learning to think about reading. **Educational Research**, 9, 56-62.

SCHMIDT,E. & JANSEN,M.(1981): **Early learnings about written language: A comparison of United States and Danish children.** Paper presented at the second European Reading Conference, Joensuu, Finland.

SOUVINEY,R.(1981): Community School Mathematics teaching curriculum inovation in Papua New Guinea. In CLARKSON,P. (Ed) **Research in Mathematics education in Papua New Guinea.** Lae, Mathematics Education Centre Press.

WILSON,B.(1981): Language and mathematics education: an international perspective. In CLARKSON,P. (Ed) **Research in Mathematics Education in Papua New Guinea.** Lae, Mathematics Education Centre Press.

2. UNIVERSITY LEVEL MATHEMATICS

WHAT PROBLEMS CAN TERTIARY STUDENTS SOLVE?

Chris Wilkins
Mathematics Department
Papua New Guinea University of Technology

Abstract

This year the Mathematics Department is running a problem solving competition for students. The questions are phrased as word problems and the solutions require algebraic manipulations, logical arguments or a combination of the two. There is a financial incentive for students to enter but otherwise participation is optional and not encouraged. Although the response to the competition was at first encouraging the demonstrated problem solving ability and/or the will of the students is not encouraging. I will detail some of the questions asked, note the frequency of correct responses, the range of answers that some students accept as reasonable, and the types of errors being made.

In previous years the Mathematics Department has attempted to run problem solving competitions for the students, and the Department has on most occasions found that the response did not justify the effort required to organise the competition. In the most successful attempt a t-shirt (with suitable inscription) was offered to students who solved most problems offered over an extended period.

This year I attempted a different style of competition. Problems are published weekly in the University information sheet - the Reporter - and a K5 cash prize is given each week to the first correct entry drawn from a barrel. As a further incentive to enter the competition, and to provide a climax later in the year, I decided to allow all weekly winners to enter a final competition, run under the same rules, with a cash prize of K50. The rules for the competition, as published in the Reporter, are included as Appendix A to this paper. Beyond the cash incentives there was no other attempt to force or persuade students to enter solutions for the problems. Participation is entirely voluntary and not encouraged, and this along with the knowledge that only a correct entry could win perhaps would lead one to expect that students would be satisfied that any entries they submitted were correct. This can rarely be assumed in assignments and tests given as part of mathematics courses, because under these conditions every student is attempting to accrue as many marks as possible.

The method or running the competition appealed because it took very little organization and was easy to administer, it gave a not inconsiderable cash incentive for students to enter each week, and it gave every student who decided to try his/her luck or skill an equal chance of winning (conditional upon getting the correct answer).

THE QUESTIONS

After consulting several Mathematics Department staff of what types of questions should be included, and then taking most notice of my own prejudices, I decided that the questions should fall into the algebraic/logic category and be phrased as word problems. I have attempted to cover a range of question types, but mindful that the questions (1) should not be trivial, (2) they should be accessible to every student at the University. This meant that problems requiring a knowledge of calculus, probability, or specialised geometry or combinatorics should not be used, (3) the questions should provide a little light relief to other readers of the Reporter. Even if only students could enter, perhaps other readers could be stimulated to attempt a few questions.

A transcript of the first 14 questions is included as Appendix B to this paper.

THE RESPONSE TO DATE

The response, although at first encouraging, has of late not been as great as I had anticipated. A quick glance at the data in Appendix C will reveal that on average approximately 20 students have entered each week, and this is in a student community of over 1000 students who have quite a few occasions indicated that an extra K5 is a necessity. But worse is what I consider to be the low proportion of correct entries among those submitted. The conclusion, and I hope that it is an erroneous conclusion, is that very few students can solve problems of the type published each week.

In Appendix C I have classified entries as being correct, near miss, or unrealistic. An example of what I would call an unreasonable answer occurred in problem 9. Here two students ran 14 km from Unitech to town and back. A popular answer had the students arriving back some 9 hours 20 minutes after they started. Assuming that the question was set and typed correctly, perhaps this answer should have been queried.

A near miss solution occurred with either a correct solution but incorrect answer (i.e. not answering the question asked), or a slight error in the logical derivation of the solution. It is not coded on how close a submitted answer is to the correct answer. For example, the answer obtained to problem 6 (where a quantity of material was purchased) by using the logic that a markup of 50% followed by a discount of 8% gives a net markup of 42% does not give a near miss answer, even though the answer obtained is very near the correct answer.

I would now like to take several of the problems and indicate where errors were made. Those who have taught post secondary students in PNG will possibly not be at all surprised by the following!

SOME OBSERVATIONS ON THE SUBMITTED ENTRIES

With the few entries submitted each week any errors that are made tend to appear in isolation and as such do not rate attention. However several errors were relatively frequent and I would like to comment on just those errors. I will work through the questions sequentially.

In question 2 (in which people at a party were offered betelnut) many entries failed to take account of the fact that in one distribution of betelnut one less student was involved. A typical entry went:

Let x be the number of students

then $3x + 2 = 2x + 8$

therefore $x = 6$

In question 3 I expected quite a few students to use the arithmetic mean instead of the harmonic mean when determining the jogging speed of the runners. In fact 26 out of 34 entries used this method and obtained the result 8.667 km/hr. The question turned out to be a trick question for the students, though it was not intended to be, nor do I think it should have been if the problem had been given some thought, and remembering that this is a technical institution.

Question 4 was a surprise. A field was described as being triangular, with a right angle, and with the longest side 3 times the length of the shortest side. Three errors were common. Most often the hypotenuse was neglected when determining the length of the longest side. Next the length of the third side was calculated by subtraction - i.e. the sides measured x, 2x and 3x km. Finally quite a few students decided that for some reason best known to themselves the triangle was isosceles - giving sides of length x, x and 3x km.

Question 6 (buying material from a shop) brought two common errors to light. One was logical - the entrants did not know how to handle the variable discount offered during the sale, the other (as mentioned above) was that the percentage markup and markdown were added instead of being multiplied to get the net markup. In retrospect I should have anticipated this last error since Business Studies students persist in making the same error even after being told explicitly that the simple arithmetic just does not work.

In the race problem (problem 9) a typical piece of logic went as follows:

> Aaron ran 1.5 km/hr faster
>
> therefore he gained 1 km every 40 minutes.
>
> It therefore would take 14 km x 40 mins = 9 hrs 20 mins
> to catch up!

Question 10 dealt with the construction of two circles from a square. It was surprising how many students took no notice of the geometry of the situation. Their solution -

> Area of square = 1m x 1m = $1\ m^2$
>
> Let radius of circle = y metres
>
> therefore $2 \times \pi y^2 = 1$
>
> i.e. $y = 0.399$ m

In the set theory problem (problem 11) where some elements of the possible cross classification were missing, very few students had any idea how to proceed. The most popular approach was to ignore most of the information and add up the data in the question directly relevant to the students who played basketball. This gives a total of 10 + 5 + 4 = 19 basketball players. The hidden 8 were missed. This error could, I suppose, be a reading error, but I would favour a lapse of logic as explanation.

SOME CONCLUDING OBSERVATIONS

Has the competition been a success? I would have to say no. It has not generated the response that I had hoped it would and it has therefore not fulfilled the main intention of a mathematics competition, which should be to encourage problem solving in the student body. It has unfortunately reinforced my, and other people's, belief that our students are not good problem solvers. I would suspect, without any proof, that many of the students would be able to solve many of the problems if the problems were set as part of a course, because in this situation the students would be keyed onto the solution technique. Without such cueing they seem to find it hard to devise a satisfactory strategy.

I should also add the hardly surprising observations that many of the correct answers week to week tended to be provided by the same students, and that there seemed to be relatively more entries form overseas students than from national students. When I was collating the answers for this paper I was surprised to find that there were not as many incorrect as I had imagined when we drew the entries each week. On many occasions we seemed to have to search through many attempts before coming to a correct solution. An analysis of the weekly winners shows them to be spread fairly uniformly over the university, with a slight preponderance of Engineering student and overseas student winners.

Which problems have proved harder? Probably the ones that require a logical deduction. The problem involving set inclusion (problem 11), while very easy if schooled in the use of Venn diagrams, is a test of logic without a formal course in set theory and proved almost impossible for the students. The time problem (problem 8) - which took me quite a few minutes to devise a solution strategy) did prove impossible. The feast problem (number 13) did not get many correct entries for what was really a very simple logic problem. A guess on my part would be that the students give up on searching for a strategy far too quickly. The straight translation - word to algebra - problem proved easier in general. The chemical mixture problem was solved relatively easily, although I do not think it easier than the straight logic problems.

Unless response to the competition falls away further I will continue it through the year (if anyone has some interesting and suitable problems I would be glad to use them). However I do not think that the response to date would warrant a rerun of the competition in future years, or at least not with the present level of monetary incentive. (It could be noted that one question was worth K10 due to a jackpot, but even this prize did not seem to present a large carrot.)

We need here at the University to encourage problem solving and diminish the students' fear of mathematics. It would seem that the competition in its present format is not achieving this goal.

APPENDIX A

1984 MATHEMATICS COMPETITION - K200 TO BE WON

The mathematics competition this year will have a different format to previous competitions. Each week for approximately 30 weeks a word problem will be published in the reporter and all students are invited to submit a mathematical solution to it. Each week the first correct answer drawn will win a cash prize of K5. No student may win more than one weekly prize. At the end of the competition the 30 weekly winners will be eligible to answer a final question, again the first correct entry opened will win, this time the prize will be K50 cash.

The Mathematics Department staff will select the problems. Although some problems may seem suited to Engineering students, or to Business Studies students, or to Surveyors, all students at Unitech should be able to answer every problem after a little thought.

The rules to the competition are:

(1) Each student may submit one entry per week. Entries must have the name, department and year of the student attached. Solutions must consist of two parts - a mathematical formulation of the word problem and its solution, and the answer to the word problem posed. (As many of the problems could be solved by trial and error an answer alone is not sufficient.)

(2) Entries must be submitted to the Mathematics Resource Centre (room MA22) by 11am on the Tuesday following publication. The draw will be made at noon and the name of the winner will be announced in the following Friday's Reporter. To guard against the possibility of some students submitting multiple entries to increase there chances of winning all entries will be scanned to ensure that the winner has submitted a single entry.

APPENDIX B

THE QUESTIONS TO DATE

Sample question (10/2/84)

A trade store owner offered tinned fish for sale at K4 per carton. One customer offered to buy all his stock for K3.75 per carton. The trade store owner replied "Usually I take one carton home for my wantoks, but your offer means that I will make one Kina more, so I will accept." How many cartons of tinned fish were there?

(Answer: 12 cartons)

1. (10/2/84)

When Markas is twice his present age he will be sixty years older than he was when he was half the age he is now. How old is Markas?

(Answer: 40 years)

2. (17/2/84)

If all but one student at a party were given 3 betelnut there would be only 2 betelnut for the last student. If instead each student had received 2 betelnut there would have been 8 left over. How many students were there at the party?

(Answer: 9 students)

3. (24/2/84)

The football coach made his players run twice around the Unitech oval at football practice. On the first lap the players jogged, and on the second lap the players had to run hard. If the running speed was exactly twice the jogging speed, and the average speed over the two laps was 14 km/hr, determine the jogging speed of the players.

(Answer: 9.75 km/hr)

4. (2/3/84)

When surveying a right angled triangular field a surprised student found that the perimeter and area were numerically equal. If the shortest side of the field was exactly one third the length of the longest side, what was the length of the shortest side? All measurements were in kilometres.

(Answer: 2 + 2 2 km)

5. (9/3/84)

A chemist pours 1 litre of water into each of two large beakers. He dissolves 3 grams of copper sulphate in the left beaker and an unknown amount of copper sulphate in the right beaker. He now pours half the contents of the left beaker into the right beaker, stirs the solution, and then pours 500 ml back into the left beaker. If as a result there are 4 grams of copper sulphate now dissolved in the left beaker, how much did the chemist initially dissolve in the right beaker?

(Answer: 6 grams)

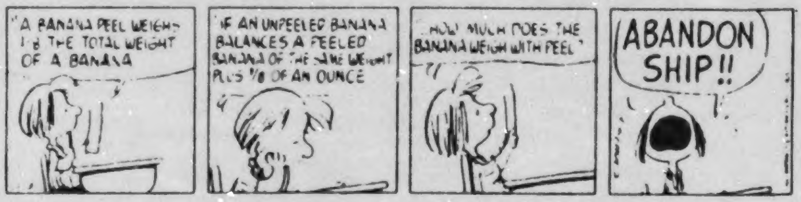
6. (16/3/84)

Burns Philp bought a large amount of material. They set their retail selling price 50% higher than their buying price. After two years they noticed that very little of the material had been sold, so they discounted the selling price by 8%. Then, during a Christmas sale, they deducted another 20 toea per metre on every metre after the first 5 metres that any customer bought. A woman bought 10 metres of this material and her bill was K19.70. What was the original price paid by Burns Philp for an 80 metre bale of material?

(Answer: K120)

7. (23/3/84)

In the cartoon below Peppermint Patti seems to be uncertain how to proceed. Can you help her?


"A BANANA PEEL WEIGHS 1/8 THE TOTAL WEIGHT OF A BANANA
IF AN UNPEELED BANANA BALANCES A PEELED BANANA OF THE SAME WEIGHT PLUS 1/8 OF AN OUNCE
...HOW MUCH DOES THE BANANA WEIGH WITH PEEL"
ABANDON SHIP!!

This week's problem is more banana madness. The only similarity to Patti's problem is that again the banana peel weighs 1/8 the total weight of a banana. Suppose that, when measured in grams, the number obtained by multiplying the weights of a banana peel and an unpeeled banana is 68 less than the number which is the weight of 20 similar peeled bananas. How much did the unpeeled banana weigh? Hint: The bananas are ripe!

(Answer: 136 grams)

8. (30/3/84)

There was a great commotion in Lower Thames Street, London, on the morning of January 12, 1887. When the early members of the staff arrived at Wapshaw's Wharf they found that the safe had been broken into, a considerable sum of money removed, and the offices left in great disorder. The night watchman was nowhere to be found, but nobody who had been acquainted with him for one moment suspected him to be guilty of the robbery. In this belief the proprietors were confirmed when, later in the day, they were informed that the poor fellow's body had been picked up by the river police. Certain marks of violence pointed to the fact that he had been brutally attacked and thrown into the river. A watch found in his pocket had stopped, as is invariably the case in such circumstances, and this was a valuable clue to the time of the outrage. But a very stupid officer (and we invariably find one or two stupid individuals in the most intelligent bodies of men) had actually amused himself by turning the hand round and round, trying to set the watch going again. After he had been severely reprimanded for this serious indiscretion, he was asked whether he could remember the time that was indicated by the watch when found. He replied that he could not, but he recollected that the hour hand and minute hand were exactly together, one above the other, and the second hand had just passed the forty-ninth second. More than this he could not remember. What was the exact time that the watch-man's watch stopped. The watch can be assumed to have been an accurate one.

(Answer: 4:21:49-1/11)

9. (6/4/84)

Aaron gave Benjamin a 10 minute start in a foot race to Ericu and back - a distance of 14 kilometres. Aaron ran 1.5 km per hour faster than Benjamin, who started at 6am sharp. Both arrived back at the same time. What time?

(Answer: 7:20 am)

10. (13/4/84)

Two equal circles are to be cut from a 1 metre square piece of plywood. What is the radius of the largest possible circles that can be cut from this timber?

(Answer: $1/(2+\sqrt{2})$ metres)

11. (20/4/84)

83 students were asked what sports they played at Unitech. 24 said that they played no sport at all, 19 said they played football only, 10 played basketball only, 5 played another sport only. 4 students said that they played both basketball and another sport (not football), and 5 played football, basketball and another sport. The remaining summary of information was lost but it is known that a total of 22 students played a sport other than football or basketball. The question is: how many students played basketball?

(Answer: 27 students)

12. (4/5/84)

Two very little girls bought two very big boxes of chocolate from Burns Philp. Each box contained hard and soft centred chocolates. The first lass ate a hard centre for each soft centre. When she had finished the soft centres there were 100 hard centres left. The second lass ate 3 hard centres for every soft centre. When she had finished the hard centres there were 100 soft centres left. How many chocolates did each box contain?

(Answer: 500 chocolates)

13. (11/5/84)

Three students decided to have a feast. One student paid K5.20 for the beer, another student paid K3.50 for some food. The third student brought K6.00 cash. All students ate and drank identical amounts. How should the K6.00 be divided among the students if all 3 are to contribute fairly to the cost of the feast.

(Answer: 2.30,0.60,3.10)

14. (18/5/84)

Three students decided to have a feast. One student brought 5 plates of food, another student brought 3 plates of food, and the third student brought K4.00 cash. All plates were of equal value and all students ate identical amounts of food. If the K4.00 brought by the third student was considered an equitable contribution for his share of the food, how should that K4.00 be divided fairly between the other two students?

(Answer: 3.50,0.50)

APPENDIX C

SUMMARY OF RESPONSES

Question	Entries submitted	Correct solutions	Near miss attempts	Unreasonable answers
1	5	5	-	-
2	23	14	-	-
3	34	2	-	2
4	27	6	-	-
5	18	16	1	-
6	21	10	-	-
7	13	8	2	1
8	4	-	1	-
9 (Jckpt)	18	7	3	8
10	11	1	2	1
11	8	2	1	1
12	7	2	1	-
13	14	3	1	-
14				

A MATHEMATICS PROFILE OF GRADE TEN LEAVERS AT THE PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY, 1984

Helen J.D. Whippy
Mathematics Department
Papua New Guinea University of Technology

The Papua New Guinea University of Technology (Unitech) accepts every year a diminishing number of school leavers from Provincial High Schools for preliminary courses in Chemical Technology, Architecture, Fisheries Diploma and Surveying. The Mathematics Department is given the task of preparing the student, in one year, to compete with students from the National High Schools who enter directly into the second year of the programme in Chemical Technology, Architecture, and Surveying.

This reports the first phase of a study using the Australian Council for Educational Research (A.C.E.R.), Mathematics Profile Series tests on operations and numbers to

1. Give a diagnostic profile of the entering Grade 10 leaver.
2. Recommend topics or teaching techniques that may be useful.
3. Gauge the students progress at mid-year and end-of-the-year.

The first phase will cover the profile of Unitech's entering Grade 10 leavers in 1984 and a discussion of the interpretation of the test results based on Australian findings.

A MATHEMATICS PROFILE OF GRADE TEN LEAVERS AT UNITECH, 1984

I. INTRODUCTION

The Papua New Guinea University of Technology accepts every year a diminishing number of school leavers from the Provincial High Schools to study a preliminary year in Chemical Technology, Architecture, Surveying and the Diploma in Fisheries. These Grade - 10 leavers are then given one year of study before they proceed to the first level of a degree course where they are expected to compete with students from the four National High Schools, who have studied for two years post-grade 10 and then entered directly into the first professional year.

Their pass/fail record in prelimary year (PY), and in the first year of the degree course is absymal. In the PY 1982, 76 students were registered, 37 of those actually passed all their subjects in that year, a pass rate of 49%. The next year, 66 entered and 34 passed - for a pass rate of 52%. (PNGUT, 1982, 1983)* The questions in the lecturer's mind is not "can it be helped?". but rather "how can it be helped?", for if there is no solution, there is no hope. So, on the premise that something positive can be done, the author elected to proceed in the following manner:

1. Do a diagnostic Test on all Grade 10 leavers entering Unitech in 1984.

2. Attempt to compare with standardized results to identify any particular strengths or weakness.

3. Recommend syllabus or methodology changes as may appear warranted in the preliminary year.

4. Retest at mid-year and end-of-the year to gauge progress.

*Compare this to an overall University pass rate of 72% combined for 1983.

II. THE INSTRUMENT

Two paper and pencil tests were selected from the Australian Council for Educational Research's (ACER) Mathematics Profile Series. One was judged to be fairly easy for the students: the Operations Tests; and one fairly difficult; the Numbers Test, Unit II.

According to the A.C.E.R. "...the Number Test consists of items designed to monitor student's development of number concepts, properties, and relationships. In conjunction with the Operations Test, which focuses on the application of these relationships to the solution of increasingly sophisticated equations, the Number Test provides for diagnosis of student understanding of concepts which underlie confident manipulations in numerical problems and those involving algebraic formulae". (Cornish and Wines, 1980)

In order to do this objectively and with some degree of confidence, the scale used is based on a probabilistic measurement model designed by George Rasch of the University of Copenhagen in the early 1960's. A few years later, Benjamin Wright of the University of Chicago developed procedures to apply the model for educational testing purposes. The unit of measurement was called a "bryte" and theoretically was to provide objectivity of measurement. The scale is based on probability, on "odds of success" in any situation.

After lengthy testing, a scale was developed for use in Australia. This will be the one referred to later when the testing results are discussed. The students ability and the item difficulty can be calibrated on the same scale, the relationship between them is illustrated in table 1 below. Notice that when the item difficulty is the same as a persons ability, the mastery level is 50% - for the odds of success are 1:1 or the probability of success is $\frac{1}{2}$.

person		MAPS ability (brytes)								
item		35	40	45	50	55	60	65	70	75
MAPS difficulty (brytes)	35	50%	75	90	96	99	100	100	100	100
	40	25	50%	75	90	96	99	100	100	100
	45	10	25	50%	75	90	96	99	100	100
	50	4	10	25	50%	75	90	96	99	100
	55	1	4	10	25	50%	75	90	96	99
	60	0	1	4	10	25	50%	75	90	96
	65	0	0	1	4	10	25	50%	75	90
	70	0	0	0	1	4	10	25	50%	75
	75	0	0	0	0	1	4	10	25	50%

Table 1: Relative mastery levels (%)

In any of the tests, each item is given a difficulty index of so many brytes. A teacher, then can select test items in the manner desired - above or below the students ability, depending on how you want to test them. The Australian system also superimposed the Piagetian operational stages on the scales to follow so that generalizations can be made concerning the progress of a student through the various Piagetian stages.

Operational stage	Mastery 25%	 50%	 75%	 90%
Early concrete operational	33	35	40	45
Late concrete operational	40	45	50	55
Early formal (concrete generalized)	50	55	60	65
Late formal operational	60	65	70	75

Table 2: Criterion abilities values (in brytes) for various mastery levels in operational stages (Cornish and Wines, 1977)

The Mathematics Profile Series can be used for individual diagnostic testing at both content and process classifications. In this case it was decided to treat the average of the cohort (n = 31) as an individual score and try to determine levels of that average.

III. RESULTS

The tests were administered during Orientation Week at Unitech in early February, 1984 to 31 students that were grade 10 leavers attending University for the first time. At a general orientation meeting for new students, they were divided into groups for discussions. The author walked off with the unsuspecting post-grade-10 group and gave them the tests before anyone could protest.

A. THE OPERATIONS TEST

Thirty-eight items were selected, although only thirty were needed. The first eight were warm-up exercises. The students were, after all, fresh in from 3 months of holidays.

Table 3 below gives the results as a percentage of the total.

			% answered		n = 31	
		Operations Tests			February, 1984	
item no.	A	B	C	D	no answers	(brytes) item difficulty
1.	6	0	[94]	0	0	26
2.	6	[94]	0	0	0	30
3.	[94]	3	0	3	0	35
4.	0	0	[100]	0	0	27
5.	0	3	[97]	0	0	37
6.	[84]	13	3	0	0	36
7.	0	[100]	0	0	0	40
8.	3	0	6	[90]	0	41
9.	3	[100]	0	0	0	39
10.	0	6	0	[94]	0	42
11.	0	[97]	3	0	0	44
12.	[97]	3	0	0	0	48
13.	0	[87]	10	3	0	47
14.	16	23	3	[58]	0	52
15.	6	[77]	6	10	0	51
16.	13	[77]	3	6	0	50
21.	3	[87]	10	0	0	35
22.	0	0	[97]	3	0	36
23.	[87]	10	0	0	3	38
24.	0	0	[100]	0	0	40
25.	0	[94]	6	0	0	37
26.	0	[84]	10	6	0	42
27.	0	0	[97]	3	0	41
28.	0	[97]	3	0	0	44
29.	[87]	10	0	0	3	43
30.	0	0	3	[97]	0	48
31.	[87]	6	3	3	0	47
32.	[77]	3	16	3	0	49
41.	0	[84]	3	6	6	37
42.	3	0	3	[87]	6	38
43.	0	10	3	[81]	6	39
44.	6	[71]	3	13	6	47
45.	[84]	6	0	3	6	42
46.	6	6	[77]	3	6	43
47.	0	77	0	16	6	45
48.	[74]	10	3	0	13	44
49.	0	42	3	[48]	6	52
50.	16	0	[74]	3	6	49

Table 3: Operations Test, 1984 (n = 31)
numbers are %.

As can be seen the students did fairly well. Items 14, 15, 16, 32, 44, 46, 47, 48, 49 and 50 are the only ones which caused any difficulty at all. These items together with their difficulty measure (in brytes) and their classification are listed below:

Item	Difficulty index (brytes)	Topic
*14	52	Small numbers
15	51	Small number
16	50	Small numbers
32	49	Large numbers
44	47	Pronumerals
46	43	
47	45	
48	44	
*49	52	
50	49	

Table 4: Items missed.

The starred items represent the two where there was approximately 50% mastery - that means the level 52 brytes would be the ability level of these students. Another table can be set up showing the levels for our student:

Mastery levels	brytes	
90%	42	
		$^{-}10$
75%	47	
		$^{-}5$
50%	52	
		$^{-}5$
25%	57	
		$^{+}10$
10%	62	

Table 5: Mastery Profile

A comparison of this table with table 3 shown the close correlation with true mastery levels (Table 3) and predicted ones (Table 5).

The preferred way of arriving at an ability level for a group however is not the method just used, but to take the average. Out of 30, $\bar{X}$ was 26.4 for this test. According to the Australian scales this represents 57 brytes (±2.9). The researcher felt this did not represent the true value as the test did not include any items of difficulty greater than 52 brytes.

The 52-bryte level puts the group in the region of late concrete operational to early formal (concrete generalized) Piagetian levels which tends to agree with other researchers. In terms of years of schooling, the Australian research puts 52 brytes at 8 going on 9 years of schooling, which is not too far off the 10 years all of the subjects had.

As a further verification of this test, if the items are ordered in increasing level of difficulty, the percentage correct should slowly decline:

Difficulty Level (brytes)	Average % correct for all items
40	100%
41	97%
42	87%
43	82%
44	89%
45	77%
47	82%
48	97%
49	80%
50	77%
51	77%
52	53%

Except for the aberration at 48 and 49 brytes, the hierarcy seems to hold.

The results here seem to indicate a strength in number work and straight forward solution of equations.

B. THE NUMBER TEST - UNIT II

There are three levels in the number test.

Test	Mean item difficulty	Suggested year range
Unit I	50.8	7-8
Unit II	54.0	8-9
Unit III	57.4	9-10

Content areas covered here are Natural numbers and integers, indices, ratios and percentages, and rational numbers. For convenient reference, the items are grouped by content area in table 6 below:

Content Area	Item no.	A	B	C	D	E	No answer	Item difficulty (brytes)	
	2	16	10	0	[61]	3	13	46	
Natural	5	0	6	[71]	13	0	10	47	
Numbers,	7	6	3	[68]	0	0	19	51	
Integers	8	0	[35]	32	13	0	19	52	
	9	0	0	19	[68]	0	13	53	
	13	23	6	[6]	3	10	52	54	
$\bar{X}_c$ = 41.3%	15	[15]	3	10	23	0	13	54	
	21	10	[6]	6	13	0	65	57	$\bar{X}_b$ = 52.5
	29	6	0	3	[6]	0	84	59	
	14	10	[29]	16	32	0	13	54	
Indices	19	[35]	6	6	6	0	45	55	
	20	[19]	23	6	6	0	48	56	
$\bar{X}_c$ = 22.25%	22	13	[6]	6	10	3	61	57	$\bar{X}_b$ = 55.5
	3	3	3	0	[84]	0	10	47	
Ratio	17	[32]	0	26	6	16	19	51	
Percentage	18	19	[35]	13	13	0	19	54	
	23	[0]	3	6	29	0	48	56	
	24	26	3	[19]	0	0	52	56	
	26	6	[26]	10	0	0	58	58	
$\bar{X}_c$ = 26.1	27	6	10	[13]	6	3	61	58	$\bar{X}_b$ = 54.8
	28	[0]	0	10	10	0	81	58	
Rational	1	3	3	[90]	0	0	3	46	
	4	19	13	0	[68]	0	0	48	
Numbers	6	10	3	[61]	13	0	13	50	
	10	29	[29]	10	3	13	16	52	
	11	13	0	6	[81]	0	0	53	
$\bar{X}_c$ = 49.9	12	3	10	45	[23]	3	19	53	
	16	3	[65]	16	3	0	13	54	
	25	3	3	3	[19]	0	71	57	$\bar{X}_b$ = 52.8
	30	0		0	3	0	84	62	

Table 6: Number Test, Unit II, % responded, n = 31, boxes indicate correct responses.

Again, after taking the odd ones out, like items 9 and 15, 23, 11 and 16; the remainder follow a similar trend. As the item difficulty increases, the number of correct responses falls. The mean on this test was 11.3 (out of 30). This corresponds to a class average of 51 brytes (±1.9), which closely parallels the result for the operations test. In terms of mastery levels:

Mastery Level	brytes
90%	41
75%	46
50%	51
25%	56
10%	61

For natural numbers and integers and rational numbers, the result is as expected. Again indicating a strength in number work.

Indices is low, indicating work is needed in this area. The same is true for Ratio and Percentage work which has the distinction of having the only two items (23 and 28) where no one got the correct answer.

III. CONCLUSIONS

The results of this testing seem to indicate that the testing does measure ability on a hierarchical scale as claimed by A.C.E.R. The aberrations however indicate that there is some difficulty in transferring the scales unchanged from Australia to Papua New Guinea. It would be tempting to state that the difficulties were due to language problems. Further research needs to be done in this area. Are the results consistent, for instance, when the Number Test is in pidgin with more familiar problems of an equivalent standard?

As for teaching material at this level, it seems to indicate a strength in number work and a need for extra work in the areas of ratio and proportion, indices, and pronumerals.

In the next stage, other areas such as measurement and time will be tested using the A.C.E.R. standardized tests for more information and another version of the Operations Test will be administered later in the year. A full report should be available early next year.

A copy of the tests is available for interested persons.

REFERENCES

Papua New Guinea University of Technology, Examination Results, 1982 and 1983. First and Second Semester.

Cornish, Greg and Wines, Robin, A.C.E.R. Mathematics Profile Series - Number Test, Australian Council for Educational Research, Ltd, 1980.

Cornish, Greg and Wines, Robin, A.C.E.R. Mathematics Profile Series - Operations Test Handbook, Australian Council for Educational Research, Ltd, 1977.

SOME CURRENT MATHEMATICAL ASPECTS IN SURVEYING EDUCATION.

P DONE*

M.A. M.Sc. F.I.Nuc.E. C.Eng. A.R.I.C. M.I.S(AUST).

K N TOMS*

M.B.E. B.Surv. B.Econ. M.U.S. Qld. F.I.S.(Aust) M.A.I.C. A.R.I.C.S. M.A.S.P.N.G.

ABSTRACT

A short outline of the mathematical principles of plane and geodetic surveying; the important statistical requirements of the student and the adjustment problem. An overview of current space techniques, with their mathematical implications. Examples are given in Appendices to the paper. (Some of these are used as student examples and/or assignments). Some pertinent questions are raised as regards mathematics education for surveyors.

CONTENTS:

1 Introduction

2 Plane Surveying

3 Geodetic Surveying

4 Mathematics Education For Established Surveying Courses

5 Conclusions

Appendices 1 - 17

*Department of Surveying & Land Studies
University of Technology
LAE
Papua New Guinea

1 INTRODUCTION

1.1 The aims of conventional surveying are to determine by appropriate observations and measurement the exact relative positions of points on the earth's surface, so that they may be represented on flat paper surfaces as maps or plans. A secondary aim is the setting out, from design data, of points to be used for the construction of man-made artificial features such as engineering works. Conventionally a system of coordinates is employed, based on three mutually orthogonal axes X,Y and Z with a convenient origin. The Z axis is considered as height, and the conventional datum for this has usually been, and predominantly still is, the so-called "MEAN SEA LEVEL", which is obtained coastally by long-term running means of sea levels measured by instruments known as tide-gauges. This last technique is a subject in itself and there are many mathematical approaches.

1.2 Any survey professionally conducted should work "FROM THE WHOLE TO THE PART" - i.e. from a set of pre-established "CONTROL" points, whose coordinates are considered to be accurately known (they are established by techniques superior to those considered adequate for the conduct of the "INFILL" ("densification") of the control, and the conduct of the actual survey itself). Thus we tend to deal with with a series of levels of known points, from "PRIMARY" through "SECONDARY" to "TERTIARY"; national survey organisations lay down rigid accuracy limits for such points. Clearly the object here is ERROR PROPAGATION CONTROL.

1.3 The principal object underlying the establishment of any such network of points is INTERNAL CONSISTENCY, both in terms of direction and in terms of distance. This is essential to any user of the ultimate product. It is in this context that survey adjustment, particularly of large networks, is of paramount importance. Adjustment is always necessary since, owing to the limitations of instrumentation, environment and human or observational techniques, small discrepancies result (for example, the sum of the observed angles of a triangle may be a few seconds of arc greater or less than 180) and these must be ironed out mathematically. "Redundant" observations also render adjustment necessary.

2 PLANE SURVEYING

2.1 This is the predominant type of 'bread and butter' surveying work. No account is taken of theoretical or actual curvature of the earth's surface and the mathematics involved are purely trigonometrical in character prior to adjustment. ANGLES are measured both vertically and horizontally by theodolites, and the use of so-called "one-second" instruments (i.e. those capable of measuring angles to 1" of arc) are very common. DISTANCES, formerly measured by tensioned invar bands, steel or cloth tapes (depending on the required accuracies) were traditionally always the surveyor's bane. At invar-band level the mathematics involved were simply those of the catenary suspension allied with appropriate temperature corrections, but in the field the practical operations were tedious and time-consuming. As a result the traditional survey method for putting in control was so called "base-extencion" by "triangulation" - only one base-line was accurately measured and a series of interconnected triangles was used to extend the network. Braced quadrilaterals were also widely employed. These have customarily been 'adjusted' in a non-rigorous way (See Appendix 1).

Latterly, since the first introduction of ELECTRONIC DISTANCE MEASUREMENT (EDM) in the 1959's, improvement of rugged, reliable, distance-measuring equipment using microwave, laser or infra-red radiation (with an elaborate, but progressively smaller, allied 'black box' to make the necessary phase comparisions between transmitted and reflected waves) has been rapid. Equipment giving digital readouts of distance to the order of mms over several km is widespread. As a result, "trilateration" has become increasingly popular, and integrated survey units capable of measuring angles and slope distances generally available; corrections are required to the raw measurements for meterological parameters (so as to attempt to account for meteorological refraction) and slope, and often these, too, can be performed by the instrument, which contains a programmed algorithm, given the necessary input.

2.2 It will be noted that refraction is a very powerful antagonist. Merely taking meteorological readings at each end of a line to obtain thereby "representative" values may in many cases be far from valid : this caveat applies both to distance measurements and to angular measurements - directions can be affected both laterally and vertically. "GRAZING" rays are particularly susceptible to refraction effects. Much research effort is directed to this aspect of survey work.

2.3 The third traditional method in plane surveying is that of "traversing", where both distances and direction are measured. The operation starts on a fixed control point (whence the bearing to another is known) and the survey progresses along a zig-zag track until it "closes" on a further (or in same cases the same) control point. The coordinates of points occupied en route are simply obtained, but invariably there is a "MISCLOSE" discovered when the computed coordinates of the end point are compared with its known coordinates. This misclose has to be "adjusted" out - i.e. distributed amongst the intermediate points so that consistency is achieved. There are many mathematical ways of doing this and the surveying profession is perpetually arguing as to which method is the "best". Rigorous or non rigorous methods may be used (Appendices 2 and 3); however, the ready availability of programmable desk calculators should result in a new outlook on established practices. Some aspects of Weighting are discussed in Appendix 13.

2.4 Traditionally, planimetric (X,Y) control networks have for many reasons tended to be developed independently from height (Z) networks. The latter are developed in a manner which is, apparently, very simple, by the use of levelling staffs and instruments known as levels - essentially telescopes whose principal axes are aligned so as to be perpendicular to the local direction of gravity at any point, this being easily achieved for example by the use of sensitive "bubbles" in glass : the Romans knew of this technique. The datum, as mentioned before, is conventionally "mean sea level". Aspects of levelling which can be very complex mathematically, and puzzling to students are, for example,: -

What is mean sea level? - This is measured coastally : coasts are notorously atypical, being in any case more subject to earth tidal loading than other places. Further, the sea is known to have its own quasi-stationary topography and slopes. The results of land-based levelling between coastal sites can pose very awkward questions both to surveyors and to oceanographers - at a point, the instrument is set up so as to be at 90 to local gravity, or in other words to be parallel to a local geopotential surface. Such surfaces are curved (broadly, they converge pole-wards). At another point, the 'level' will align itself again with the local equipotential surface. "Level" planes are not "horizontal" - and this leads to a number of interesting definitions whereby "differences in height" may be measured. Conceptually it is evident that a knowledge of gravity and geopotential fields is essential in this aspect of surveying, but few practising surveyors need to get so closely involved with this theory, since the bulk of their work concerns relatively small areas over which geopotential surfaces may be considered as both level and horizontal. For a discussion of height concepts, see Appendix 4.

2.5 Adjustments to surveying networks (whether planimetric, or one dimensional height networks) are always necessary. This is because, as indicated above, observations contain inherent errors, and also because prudent survey practice involves redundant observations - e.g. points are coordinated by observations from a number of stations and the results vary. The task of adjustment computations is to "adjust" the discrepancies so that the adjusted quantities are as close as possible to the (unknown) "true" value. The rigorous form of adjustment is that developed by Gauss in 1794; it has the advantage of being non-arbitrary, it yields sensible and consistent values for the adjusted quantities and it has a sound statistical and mathematical basis. It is known as "least squares" - each observation has a residual v added to it and the method is aimed at minimizing the squared sum of these quantities:-

Σ(v v) --> minimum, or Σ(pvv) --> minimum

(The weight p is often the reciprocal of the variance of the observed quantity). Least squares adjustments involve a close familiarity with matrix algebra, "condition" or "observation" equations may be involved, and the matrices may be very large. For an example of the use of the technique for trilateration and intersection, see Appendix 5 and for a simple system of levels see Appendix 6.

The statistical needs of the surveying student are very important, and include a basic knowledge of the normal distribution, errors, their analysis and propagation (of both systematic and random types), rejection criteria and confidence levels. Sampling theory, X^2 and f testing are also a necessary part of the professional surveyor's armoury.

3 GEODETIC SURVEYING

3.1 This concerns itself with the larger stage : the earth is curved, so that a system of primary geodetic control over a large area, such as a country or a continent, must take this into account. Since the geoid, or mean sea level, surface is irregular, a theoretical surface, an ellipsoid, or oblate spheroid, is postulated, and the coordinates of points are referred to this, so that instead of (X Y Z) coordinates we now have (ϕ,λ ,h) with ϕ being geodetic latitude, λ being geodetic longitude, and h being height above this entirely imaginary surface. The space techniques described below give such coordinates. Computations now become more complex, because they are concerned with observed quantities being firstly "reduced" to this ellipsoid, (or "spheroid") and then fairly involved formulae bound up with the doubly curved geodesic, the shortest distance between two points on this surface, are invoked. Computations involve the "forward" and "reverse" problems: these together with allied computations are covered in Appendices 7 and 8.

3.2 The final stage, that of projecting the points on to plane paper, is also a specialised business, since this may be done in various ways. Conic, cylindrical and azimuthal projections are possible. The resulting map may be such that all directions are preserved i.e. the shape of a figure on the map will be similar to that on the spheroid (conformal), or that the ratio between various areas is unchanged (equal area), or that certain distances are projected without length distortion (equidistant) or that none of these properties exist, in which case the term general is used. So far as map production is concerned it is possible to distinguish between geometric, semi-geometric and mathematical projections. In the first case the projection is performed through geometrical means, such as perspective construction, in the second, points situated along certain lines only (e.g. meridians) are projected geometrically, and in the third the relationship between the ellipsoidal coordinates and the corresponding map coordinates is purely analytical. Many aspects of map projections involve more material than could be conveniently summarised in a brief Appendix: details may be found in MATHER, R.S. The theory and use of some common Map Projections. Univ. of NSW. Monograph 1. (125 pp).

3.3 Papua New Guinea is a country notoriously inimical to any form of conventional surveying - rugged terrain, a hostile climate and ecology are obvious factors. It is not surprising that its geodetic development has been very limited. Perhaps it is simplistic to state that here technology works in reverse - for example, generally, the aeroplane preceded 'the wheel' and video machines have a lead on television. Similarly, in geodetic surveying, it seems that any future integrated control base will be one generated from space technology, which has developed considerably in recent years. (The Department of Surveying here at Unitech conducts a course in satellite surveying and is currently involved in LANDSAT research).

3.4 The greatest single influence on geodetic theory and operations has been the introduction of artificial satellites. Observing these satellites has resulted in improved knowledge of the earth's gravity field and increased accuracy of relative locations; for example, relative positions of widely separated points can now be accurately determined in a common coordinate system. Such accuracies are the result of observations on the Navy Navigation Satellite System (NNSS), which depend on the change in frequency (i.e. Doppler shift) of a transmitted radio signal and improved frequency (time) standards. Like electronic distance measurements, the Doppler signal must be corrected for propagation delay due to the atmosphere; therefore, the NNSS transmits the frequency pair 150 and 400 MHz by which the discrepancy can be resolved, resulting in measurement errors associated with a single pass of this Doppler system of less than 5m. The frequency shift rapidly goes to zero as the line of sight becomes perpendicular to the path of the satellite.

The further the satellite is from the observer (receiver), the more gradual is this frequency shift. Consequently, the rate of change of frequency determines the relative position of the satellite and observer (receiver). Knowing the position of the satellite as a function of time - its ephemeris - the observed Doppler data fix the position of the receiver in the same coordinate system. Although not designed for accuracies comparable with those achieved in geodetic surveying, the NNSS allows the determination of relative positions of two sites with a precision of better than 1 m. To attain such accuracy requires many satellite passes and many observations in order to average out random errors. A simple treatment of the Doppler theory is given in Appendix 9.

3.5 Determination of the ephemeris of a satellite involves a sequence of modelling the earth's gravitational field and comparing the position calculated with the modeled field to the actual position of the satellite. The combination of data resulting from tracing many satellites leads to highly accurate gravity models, such as the Goddard Earth Model named after the NASA center where the data are processed. Only recently has the spatial resolution of satellite-determined gravity become reliable for wavelengths shorter than approximately 1/15 of the circumference of the earth, and as local gravity measurements are sensitive to the mass attraction of local terrain features, it has been difficult to bring together surface gravity data and satellite data in a complementary way, although there is progress toward this goal. In particular, the use of recent satellite altimetry has dramatically improved the knowledge of gravity at shorter wavelengths over oceans.

3.6 A radar altimeter on a satellite measures the time of transit of a pulse of electromagnetic energy from the satellite to the surface of the ocean, whence it is reflected back to the satellite. The spherical shape of the earth causes the return energy to be spread out. If the ocean were perfectly smooth, the return signal would be sharp at its leading edge, but waves on the ocean surface disperse the energy of the returning signal, and the observer must decide where the leading edge of the signal actually is. It should be noted that the slope of this returning signal is of significant interest to oceanographers and vessel operators for it is related to the local sea state beneath the satellite. Altimeters have flown on Skylab, and an improved altimeter with a 10 cm sensitivity has been used on SEASAT 1.

3.7 The Global Positioning System's (GPS) planned eighteen 12-hour revolution satellites in three orbital planes will afford continuous coverage of the earth. The system is designed primarily for a navigational accuracy of from 10 to 15 m. The accuracy can be attained by placing highly accurate clocks based on a caesium or rubidium standard in all the satellites. A coded train of pulses emitted from four satellites will determine the position of receiver by the relative time of arrival of the signal and will also enable the receiver to determine time with great accuracy. Improved clocks using a hydrogen maser, longer averaging times than the 1-sec reading used in navigation, and employment of the Doppler information intrinsic in the signals but not needed for navigation might permit determination of positions on the ground to a small fraction of a meter. Such an improved GPS would provide geodesists with a powerful tool for the classification of geodetic control networks. A discussion of GPS theory is given in Appendix 10.

3.8 A Doppler Satellite technique which has been used successfully in Papua New Guinea is that called translocation. The simplest form of translocation may be visualized by considering two stations tracking the same satellite pass. If point positions are computed for both stations, they will contain the same systematic orbital errors because of the commonality of data. Even though the point positions of both stations are fully affected by the systematic errors in the satellite orbit, the interstation vector will be relatively error free. If one of the stations has a known position (e.g. a trigonometrical station in a national network) the differences between the known coordinates and those obtained from point positioning may be calculated. The differences may then be applied to the point positioning coordinates of the second station to obtain a relatively accurate position for that station. This technique is often referred to as displacement translocation. The method relies on the systematic orbital errors "looking the same" from both stations and will only work well where each satellite passes to the same side of both stations. If the satellite passes between the two stations, the orbital errors will have differing effects on the two point positions, and the inter-station vector will no longer be as free of the orbital errors. It should be noted that displacement translocation as described above does not involve any mathematical model for orbital adjustment, it simply involves shifting the computed ground station coordinates to fit a known position. When Doppler results (ϕ, λ, h) are to be used in a local or national scheme of coordinates (X,Y,Z), datum transformations are needed. Appendix 11 gives some details on this subject.

4 MATHEMATICS EDUCATION FOR ESTABLISHED SURVEYING COURSES.

4.1 The preceding sections provide an overveiw of modern professional surveying activity. Appendices 1 - 14 give some indication of the application of mathematics to the theory of surveying science. The question that poses itself is how to provide in secondary and tertiary curricula the mathematical education on which a full understanding of the principles and practice of surveying can be based?

An example of professional surveying course design is given in Appendix 15. This sets out the structure of the University of Queensland Bachelor of Surveying program at pass and honours level together with the syllabi for the mathematics components taught as service subjects by the Department of Mathematics.

In addition to the latter, the subjects shown in table 1 below are offered "inhouse" by the Department of Surveying.

TABLE 1

SUBJECT	SYLLABUS
SV115 SURVEYING MATHEMATICS I	Plane, solid Euclidean geometry, trigonometry. Geometric, trigonometric analysis. Principles, techniques of computation. Application to typical survey problems.
SV216 SURVEYING MATHEMATICS II	Algebra of vectors: spherical trigonometry, straight lines & planes in 3 dimensions, applications in spherical astronomy, photogrammetry. Differential vector calculus: properties of surface curves and surfaces, applications in geometric geodesy.
SV112 PROJECTIVE GEOMETRY & MATRIX ALGEBRA.	Elementary properties of space & plane perspective transformation. Geometrical applications of matrix algebra. Applications to typical photogrammetric problems.

SV350 GEODETIC MATHEMATICS	Integral vector calculus, orthogonal curvilinear co-ordinates. Green's identities, harmonic functions. Dirichlet's principle, applications to physical geodesy.

The general basic mathematics subjects in Appendix 15 (ME100, ME101, ME102, ME103, ME104 AND ME203) together with the specialised units contained in Table 2 above attract 69 out of a total of 285 credit points (25%) for the pass degree taken over six semesters.

Entry to the Queensland Bachelor of Surveying for Grade 12 school leavers is subject to a prescribed level of achievement as reflected in the statewide assessment for the Tertiary Entrance Score. In addition certain Grade 12 subjects are specified as prerequisites. These include Mathematics. While not identical to those applying in other states the Queensland course is reasonably representative of Surveying degree programs in terms of entry requirements and course design. It is pertinent to note here that all Surveying degree courses are subject to scrutiny and accreditation by the various Australian and New Zealand registration authorities through the medium of the Conference of Reciprocating Surveyors Boards of Australia and New Zealand.

4.2 In Papua New Guinea tertiary education for Surveying is provided through the medium of the Papua New Guinea University of Technology two-tiered program leading to the Diploma/Bachelor of Technology in Surveying (see Appendix 16). Entry to this course pattern at Grade 12 level requires passes at C level in Mathematics (major), Science and English. The Diploma in Surveying is taken over four semesters of full-time study and one-year of practical experience in industry. A diplomate may apply for entry to the professional year (2 semesters of full-time study). Admission is not automatic.

For reasons associated with the level of Grade 12 in the secondary school system in Papua New Guinea and course duration constraints applying in the University, no claim to parity with the Queensland program is made. A further factor acting against parity is the comparative paucity of mathematics material in the program (see Appendix 17). This consists of 15 semester hours per week out of a total of 150 (15%).

5 CONCLUSIONS

5.1 In this very short review there have been obvious omissions and all facets of this mathematically-based discipline have not perforce been covered. Obvious absentees have been astronomy, much physical geodesy and spherical harmonic relationships for the earth's geopotential field, the mathematics of satellite orbits, with their Keplerian elements and the significance of their various rates of change, LANDSAT, engineering surveying, cadastral work, very long base line interferometry (V.L.B.I. - mentioned only in passing in Appendix 10), photogrammetry, digitisation of survey data and the subject of land 'data banks'. Many new techniques are emerging and the mathematical and computer aspects of the subject are becoming more and more demanding. An attempt has been made to show that perhaps the land surveyor should not necessarily be regarded by academic theoreticians as a technician whose proper place is the bush and whose mathematical education is negligible. Surveying offers a challenging career to those who are lucky enough to be both mentally very alert, physically reasonably fit and prepared to spend some time away from their families.

5.2 Section 4 emphasises the relative paucity of mathematical material taught to surveyors at P.N.G.U.T. as compared with a reasonably typical Australian institution. Attempts to meet the disparities between what is taught and what is needed in this increasingly technological discipline cannot really be made up in-house, if, as is obviously considered desirable, an aim is parity of recognition. It would seem obvious that an extra year, devoted *inter alia* to Mathematics, Computing, Physics and English must be seriously considered. Again, curricula designers at secondary level could be requested to consider the needs of Surveying when drawing up courses which lay the foundation for tertiary studies.

APPENDIX 1

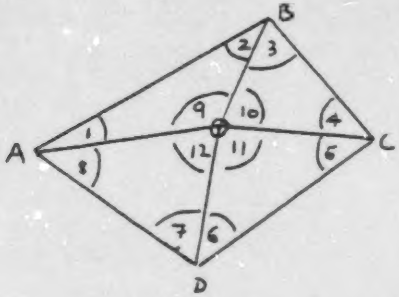
NON RIGOROUS ADJUSTMENT OF TRIANGULATION

APPENDIX 1

NON RIGOROUS ADJUSTMENT OF TRIANGULATION

Quadrilaterals, either braced or with a central station, usually form the basic elements of triangulation schemes. They may be adjusted by a method which involves condition equations and which derives mutually consistent adjusted angles: the method is known as that of "equal shifts".

QUADRILATERAL WITH AN INTERNAL STATION



If we were to start at A, with one distance AB, only 6 angles would be needed in theory to "fix" the figure.

An extra observed angle must therefore produce an added "redundancy" and therefore an added condition equation.

In the fully observed figure we have 12 angles so there must be 6 redundant observations and 6 condition equations

1. "ANGLE" equations

$$1 + 2 + 9 = 180°$$
$$3 + 4 + 10 = 180°$$
$$5 + 6 + 11 = 180°$$
$$7 + 8 + 12 = 180°$$

2. "STATION" equation

$$9 + 10 + 11 + 12 = 360°$$

3. "SIDE" condition

$$\frac{OA}{\text{Sin } 2} = \frac{OB}{\text{Sin } 1} : \frac{OB}{\text{Sin } 4} = \frac{OC}{\text{Sin } 3} \cdot \frac{OC}{\text{Sin } 6} = \frac{OD}{\text{Sin } 5} \cdot \frac{OD}{\text{Sin } 8} = \frac{OA}{\text{Sin } 7}$$

$$\therefore \frac{OA}{OB} . \frac{\text{Sin } 1}{\text{Sin } 2} . \frac{OB \text{ Sin } 3}{OC \text{ Sin } 4} . \frac{OC \text{ Sin } 5}{OD \text{ Sin } 6} . \frac{OD \text{ Sin } 7}{OA \text{ Sin } 8} = 1$$

$$\therefore \text{Sin } 1. \text{ Sin } 3. \text{ Sin } 5. \text{ Sin } 7 = \text{Sin } 2. \text{ Sin } 4. \text{ Sin } 6. \text{ Sin } 8$$

or Σ (log Sins) (odd angles) = Σ (log Sins) (even angles)

Since the actual observations are not "perfect" the condition equations will not be satisfied merely by inserting the observed values. However if we assume that each observation is accorded a small, unknown correction, v, the equations can be satisfied.

The first equation becomes

$$1 + v_1 + 2 + v_2 + 9 + v_9 = 180$$

or, $v_1 + v_2 + v_9 = 180 - (1+2+9) = C_1$

Where C_1 is the misclosure

Similarly, $v_3 + v_4 + v_{10} = 180 - (3 + 4 + 10) = C_2$

$v_5 + v_6 + v_{11} = 180 - (5 + 6 + 11) = C_3$

$v_7 + v_8 + v_{12} = 180 - (7 + 8 + 12) = C_4$

$v_9 + v_{10} + v_{11} + v_{12} = 360 - (9 + 10 + 11 + 12) = C_5$

The side equation becomes:

(using the abbreviation "LS" = log sin)

$$\begin{bmatrix} LS \\ (1+v_1) \end{bmatrix} - \begin{bmatrix} LS \\ (2+v_2) \end{bmatrix} + \begin{bmatrix} LS \\ (3+v_3) \end{bmatrix} - \begin{bmatrix} LS \\ (4+v_4) \end{bmatrix} + \begin{bmatrix} LS \\ (5+v_5) \end{bmatrix} - \begin{bmatrix} LS \\ (6+v_6) \end{bmatrix} + \begin{bmatrix} LS \\ (7+v_7) \end{bmatrix} - \begin{bmatrix} LS \\ (8+v_8) \end{bmatrix} = 0$$

This may be approximated, to give

$$d_1 . v_1 - d_2 . v_2 + d_3 . v_3 - d_4 . v_4 + d_5 . v_5 - d_6 . v_6 + d_7 . v_7 - d_8 . v_8 + c_6 = 0$$

Where

d_i = change in log Sin i for change of 1" in angle i

C_6 = log Sin 1 - log Sin 2 + log Sin 3 - log Sin 4 + log Sin 5 - log Sin 6 + log Sin 7 - log Sin 8.

There are 6 equations with 12 unknowns (v_1 to v_{12}) so there are ∞ solutions possible.

BEST COPY AVAILABLE

BEST COPY AVAILABLE

However, if we impose the least squares condition.

$$\Sigma v^2 \rightarrow \text{minimum},$$

a unique solution will result. However, unless a computer is available, too much labour is involved in doing the matrix calculations.

The approximate (semi-rigorous) solution yielded by the equal shifts method involves satisfying the conditions successively rather than simultaneously. Therefore:-

1. The method is only suitable for simple figures - otherwise it becomes too complex;
2. The method is only approximate - a different result is obtained if the order of procedure is changed.

EXAMPLE

Figure as on page 1.

ANGLE	°	'	"
1	26	17	20
2	38	53	54
3	43	11	37
4	21	30	36
5	103	29	22
6	50	03	41
7	27	05	17
8	49	27	41
9	114	48	33
10	115	17	49
11	26	26	38
12	103	27	00

OBSERVED ANGLES	2 CORRECT FOR MISCLOSE	CORRECTED ANGLE	4 CONDITION REBALANCE	5 CORRECTED ANGLE	6 SIDE CORRECTION	FINAL ANGLE
Δ ABO						
1 26° 17' 20"	+5	25	+1	26	-10.9	26° 17' 15.1"
2 38 53 54	+4	58	+1	59	+10.9	38 54 09.9
9 114 48 33	+4	37	-2	35		114 48 35
179 59 47						
+ 13						
Δ BCO						
3 43 11 37	-1	36	+2	38	-10.9	43° 11 27.1"
4 21 30 36	0	36	+1	37	+10.9	21 30 47.9
10 115 17 49	-1	48	-3	45		115 17 45
180 00 02						
-02						
Δ CDO						
5 103 29 22	+6	28	+1	29	-10.9	103° 29 18.1
6 50 03 41	+6	47	+1	48	+10.9	50 03 58.9
11 26 26 38	+7	45	-2	43		26 26 43
179 59 41						
+ 19						
Δ ADO						
7 27 05 17	+1	18	+2	20	-10.9	27° 05 09.1"
3 49 27 41	+1	42	+1	43	+10.9	49 27 53.9
12 103 27 00	0	00	-3	26 57		103 26 57
179 59 58						
+ 02						
Δ STATION						
9 114 48 37	-2	35				
10 115 17 48	-3	45				
11 26 26 45	-2	43				
12 103 27 00	-3	57				
306 00 10						
- 10						

NOTE: 1. After column 2, the initial balance is upset. A further correction is made in column 4.

2. Following column 5 completion, go to side condition table P.T.O.

BEST COPY AVAILABLE

SIDE CONDITION

	ANGLE			LOG SIN	1" DIFF		ANGLE			LOG SIN	1" DIFF
	°	'	"	.	.000 00		°	'	"	.	.000 00
1	26	17	26	64632864	426	2	38	53	59	79793149	261
3	43	11	38	83535401	224	4	21	30	37	56427315	534
5	103	29	29	98784718	-051	6	50	03	48	88465633	176
7	27	05	20	65836657	412	8	49	27	43	88079898	180
			Σ	12789640	1011				Σ	12765995	1151
			-D	12765995	1151	+E				D	E
			+A	23645	2162	B					

CORRECTION TO EACH ANGLE $= \pm \frac{A''}{B}$

NOTE:-

The condition equation becomes:-

$426v_1 - 261v_2 + 224v_3 - 534v_4 - 51v_5 - 176v_6 + 412v_7 -$
$180v_8 + 23645 = 0$

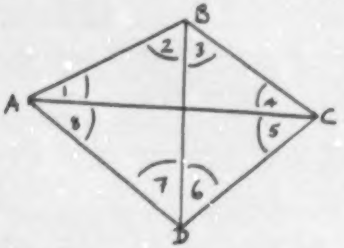
CORRECTION TO EACH ANGLE = ± 10".9
1 3 5 7 - - 10".9 (Now return to column 6)
2 4 6 8 - + 10".9

Note: if A is -ve correction to 1 3 5 7 is +
2 4 6 8 is -

if A is +ve correction to 1 3 5 7 is -
2 4 6 8 is +

BRACED QUADRILATERAL

Here 8 angles are observed but there is no central station:-



Since only 4 angles are required, in theory, to fix the figure, there will be 4 independent condition equations.

ANGLE equations:-

(Δ ABC) $v_1 + v_2 + v_3 + v_4 = 180° - (1+2+3+4) = C_1$
(Δ CDA) $v_5 + v_6 + v_7 + v_8 = 180° - (5+6+7+8) = C_2$
(Δ ABD) $v_1 + v_2 + v_7 + v_8 = 180° - (1+2+7+8) = C_3$

SIDE equation:-

$d_1 v_1 - d_2 v_2 + d_3 v_3 - d_4 v_4 + d_5 v_5 - d_6 v_6 - d_7 v_7 +$
$d_8 v_8 + C_4 = 0$

EXAMPLE
The observed values are:-

1	84	08	00"	5	75	03'	02"
2	40	10	36	6	49	15	28
3	16	11	20	7	30	05	21
4	39	29	53	8	25	36	25

1 OBSERVED ANGLES	2 CORRECTION FOR MISCLOSURE	3 CORRECTED ANGLE	4 OVERLAP CORRECTION	5 CORRECTED ANGLE	6 SIDE CORRECTION	7 FINAL VALUE
Δ ABC						
84 08 00	+2	02	-4	07 58	+2".5	84° 08'00".5
40 10 36	+3	39	-5	10 34	-2.5	40 10 31.5
16 11 20	+3	23	+4	11 27	+2.5	16 11 29.5
39 29 53	+3	56	+5	30 01	-2.5	39 29 58.5
179 59 49						
+ 11						
Δ CDA						
75 03 02	-4	02 58	+5	03 03	+2.5	75° 03'05".5
49 15 28	-4	24	+5	15 29	-2.5	49 15 26.5
30 05 21	-4	17	-5	05 12	+2.5	30 05 14.5
25 36 25	-4	21	-5	36 16	-2.5	25 36 13.5
180 00 16						
- 16						
Δ ABD						
84 08 02	-4	07 58				
40 10 39	-5	34				
30 05 17	-5	12				
25 36 21	-5	16				
180 00 19						
- 19						

NOTE: 1. After column 2, the initial balance is upset. An overlap correction is then made in column 4.

2. After column 5, go to side condition table to obtain correction (page 9).

SIDE CONDITION

	FROM COL 5 ANGLE			LOG SIN	1" DIFF		FROM COL 5 ANGLE			LOG SIN	1" DIFF
	°	'	"		.000 00		°	'	"	.	.000 00
1	84	07	58	99771095	22	2	40	10	34	80965341	249
3	16	11	27	44535113	725	4	39	30	01	80351308	255
5	75	03	03	98504684	56	6	49	15	29	87947250	181
7	30	05	12	70010584	363	8	25	36	16	63564025	439
			Σ	12822276	1166				Σ	12827924	1124
			-D	12827924	1124	+E				D	E
			-A	5648	2290	B					

CORRECTION TO EACH ANGLE = ± $\frac{A''}{B}$

The condition equation becomes:-

$$22 v_1 - 249 v_2 + 725 v_3 - 255 v_4 - 56 v_5 - 181 v_6 + 363 v_7 - 439 v_8 - 5648 = 0$$

CORRECTION TO EACH ANGLE = ± 2.5"

1 3 5 7 - + 2".5

2 4 6 8 - - 2".5

6

Note: if A is - correction to 1 3 5 7 is +

2 4 6 8 is -

if A is + correction to 1 3 5 7 is -

2 4 6 8 is +

APPENDIX 2

RIGOROUS TRAVERSE ADJUSTMENT

In order to satisfy the principle of least squares the conditions of a traverse adjustment may be expressed as follows:

(i) the sum of the weighted squares of the residuals must be a minimum;

(ii) the residual errors in the eastings of the lines must sum to the misclose in eastings (e_t);

(iii) the residual errors in the northings of the lines must sum to misclose in northings (n_t);

(iv) the residual errors in the angles at the stations must sum to a_T the angular error of closure of the traverse.

Two approaches to the problem are described.

METHOD A

Weights. It is necessary to define the relative weights of linear and angular measurements. Assuming that the distances have been measured by EDM then the linear error ratio can be described in the form:

$$\text{Linear error ratio} = \frac{\text{Standard error of measurement of line of distance } S}{S}$$

With most EDM instruments, this becomes:

$$\text{Linear error ratio} = \frac{Z + SF\ 10^{-6}}{S}$$

where Z = zero error, F = fractional error.

In order to express the angular error in a similar fashion it has first to be translated into a transverse distance which is dependent on the length of the line.

One way in which the angular error at a station can be determined is to divide the actual angular error of closure by (no of angles)$^{\frac{1}{2}}$. An alternative, which is probably more reliable, is to estimate the standard error of an angle from analysis of previous results obtained in a comparable class of traverse. Then, for a line of length S and an angular error a, the proportional transverse error is given by:

$$\text{Transverse error ratio} = \frac{S \sin a}{S} \text{ or } a'' \sin 1''.$$

$$\text{The ratio } \frac{\text{linear error ratio}}{\text{angular error ratio}} \text{ becomes } \frac{Z + SF\ 10^{-6}}{Sa'' \sin 1''} = K.$$

Thus, K is a measure of the relative effect of linear and angular errors. According to the theory of errors, weight is inversely proportional to the square of sigma and, therefore, in the least squares condition equation those elements which represent the linear residual errors should be weighted by the factor $\frac{1}{K^2}$, whilst those elements which represent the angular residual errors should have a weight of unity.

If the bearing of a line from Stn m of a traverse is altered by a small amount, b, then the position of all subsequent stations is altered as a consequence. Figure 1 shows the effect of such a change, at Stn m, on the position of the terminal point Stn n, S' being the length of the line joining m and n, B being the calculated bearing of that line. Resolving the change in position of Stn n into its components, we have:

Change in latitude $= -S'b \; Sin \; B = -b(E_{Tn} - E_{Tm})$

Change in departure $= +S'b \; Cos \; B = +b(N_{Tn} - N_{Tm})$

If the same quantity is subtracted from all northings of stations then changes in northings are unaffected. Similar logic applies to the eastings. Therefore, subtracting N_{Tn} from all latitudes and E_{Tn} from all departures, namely, making Stn n the coordinate origin of the traverse, the above changes in the position of Stn n may be expressed as:

Change in latitude $= +bE_{Tm}$ (Changes due to change

Change in departure $= -bN_{Tm}$ in bearing at Stn m)

Also, a change in length s, in a line of length S joining Stn m and Stn $(m + 1)$ produces component changes in the position of Stn $(m + 1)$ and these changes affect the position of all stations beyond $(m + 1)$ by the same amount. Thus:

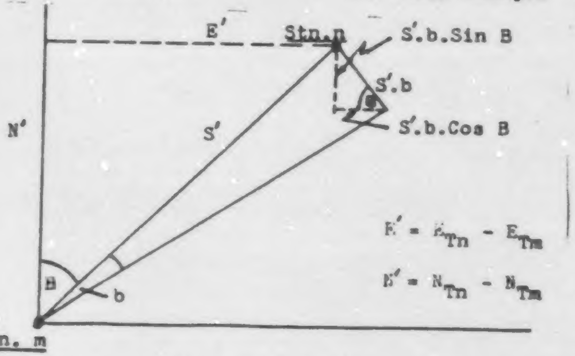


Figure 1 Effects resulting when a bearing on a line from station m is adjusted by amount b

Change in latitude at Stn $n = \frac{sN}{S}$ (Changes due to change

Change in departure at Stn $n = \frac{sE}{S}$ in length of line from Stn m to Stn $(m + 1)$)

where E and N are the respective partial coordinates of line m to $(m + 1)$.

Therefore, the total change in the position Stn n as a result of a small change in bearing b, and a small change in distance s, of a line from an intermediate station of a traverse (Stn m) is given by:

Change in departure $= -bN_{Tm} + \frac{sE}{S}$

Change in latitude $= +bE_{Tm} + \frac{sN}{S}$

The conditions for the adjustment are therefore:

(i) $\Sigma b^2 + \Sigma \frac{1}{K^2} \left(\frac{s}{S}\right)^2 =$ a minimum

(ii) $\Sigma -bN_T + \Sigma \frac{s}{S}E = e_T$

(iii) $\Sigma bE_T + \Sigma \frac{s}{S}N = n_T$

(iv) $\Sigma b \qquad = b_T$

In equation (iv) $b_T = 0$ providing that the original angular error of closure has been distributed amongst the observed angles. If this preliminary adjustment has not been made, then b_T is equal to the original closing correction.

The determination of b at each station and of s for each line is achieved by the method of correction.

The determination of b at each station and of s for each line is achieved by the method of correlative normal equations, the derivation of which is as follows:

Let $\frac{s}{S} = M$. Therefore $s = SM$.

$$b_1^2 + b_2^2 + b_3^2 \ldots + \frac{M_1^2}{K_1^2} + \frac{M_2^2}{K_2^2} + \frac{M_3^2}{K_3^2} \ldots = a \text{ min}$$

$$-b_1 N_{T_1} - b_2 N_{T_2} - b_3 N_{T_3} \ldots + M_1 E_1 + M_2 E_2 + M_3 E_3 \ldots\ldots = e_T$$

$$b_1 E_{T_1} + b_2 E_{T_2} + b_3 E_{T_3} \ldots + M_1 N_1 + M_2 N_2 + N_3 N_3 \ldots = n_T$$

$$b_1 + b_2 + b_3 \ldots = b_T$$

Introducing the correlatives in the usual way gives:

$$b_1 = -C_1 N_{T_1} + C_2 E_{T_1} + C_3; \quad M_1 = K_1^2 (C_1 E_1 + C_2 N_1)$$

$$b_2 = -C_1 N_{T_2} + C_2 E_{T_2} + C_3; \quad M_2 = K_2^2 (C_1 E_2 + C_2 N_2)$$

and so on.

Substitution of these in the condition equations gives:

$$N_{T_1}(C_1 N_{T_1} - C_2 E_{T_1} - C_3) + N_{T_2}(C_1 N_{T_2} - C_2 E_{T_2} - C_3)\ldots$$
$$+ E_1 K_1^2 (C_1 E_1 + C_2 N_1) + E_2 K_2^2 (C_1 E_2 + C_2 N_2)\ldots = e_T$$

$$E_{T_1}(-C_1 N_{T_1} + C_2 E_{T_1} + C_3) + E_{T_2}(-C_1 N_{T_2} + C_2 E_{T_2} + C_3)$$
$$+ N_1 K_1^2 (C_1 E_1 + C_2 N_1) + N_2 K_2^2 (C_1 E_2 + C_2 N_2)\ldots = n_T$$

$$(-C_1 N_{T_1} + C_2 E_{T_1} + C_3) + (-C_1 N_{T_2} + C_2 E_2 + C_3) + \ldots + C_3 = 0 \text{ (or } b_T)$$

The normal equations from which the values of C_1, C_2 and C_3 are obtained are:

$$C_1 [\Sigma N_T^2 + \Sigma K^2 E^2] + C_2 [\Sigma K^2 NE - \Sigma E_T N_T] - C_3 [\Sigma N_T] = e_T$$

$$C_1 [\Sigma K^2 EN - \Sigma E_T N_T] + C_2 [\Sigma E_T^2 + \Sigma K^2 N^2] + C_3 [\Sigma E_T] = n_T$$

$$-C_1 [\Sigma N_T] + C_2 [\Sigma E_T] + nC_3 = 0 \text{ (or } b_T)$$

The values of C obtained from the above are then used to calculate b for each station and $s(SM)$ for each line, the adjusted bearings and distances being used to give the final coordinates.

An HP 97 program for this method may utilise two tapes; the use of the Standard pac "3 x 3 matrix operations" is convenient for the solution of the normal equations. Further, the method lends itself to two approaches. In the first, the value of K is computed for each individual line, while in the second, the value of K is the same for all lines of the traverse.

As implied earlier, where linear measurements are obtained by steel band one may adopt a value of K which is the same for all lines on the assumption that the precision of linear measurements is the same for all lines. For example, if one assumes a standard error of ±5 sec for angles and a linear accuracy of 1 in 12000 then K will have a value of 3.5 approx.

If EDM measurements are used the situation changes. For example, in the case of the AGA 12 the manufacturer quotes an mse in the form: ±5 mm + 10 ppm (ie Z = 5 mm, F = 10 parts per million of the distance). Such a statement is itself open to question, but for a distance S the implication is that the mse is $\pm(Z + SF)$ and, for the purposes of this paper, this is the interpretation used. On this basis, assuming an angular accuracy of ±5 sec, the values of K for various distances S, are as in table 1.

S(metres)	K
20	10.7
50	4.5
100	2.5
150	1.8
200	1.4
300	1.1
400	0.9
500	0.8
1 000	0.6

Table 1.

It can be seen that K changes rapidly in the first 400 m range and this includes a high proportion of the distances involved in mine traverses. For this reason it is advisable to use the first method since the variable K is more likely to result in a realistic adjustment.

METHOD B

The derivation in this method is based on a more direct approach. If the length S of a line is changed by a small amount s and the bearing B simultaneously altered by a small amount b, then the condition for closure in the latitudes of a traverse is:

$$\Sigma(S + s)\ \mathrm{Cos}\ (B + b) = N_T,$$

the difference in total latitude of the initial and closing stations.

Expanding this, we have:

$$\Sigma(S+s)(\mathrm{Cos}\ B\ \mathrm{Cos}\ b - \mathrm{Sin}\ B\ \mathrm{Sin}\ b) = N_T$$

Therefore

$$\Sigma(S+s)(\mathrm{Cos}\ B - b\ \mathrm{Sin}\ B) \simeq N_T \quad (b \text{ in radians})$$

Therefore

$$\Sigma(S\ \mathrm{Cos}\ B - bS\ \mathrm{Sin}\ B + s\ \mathrm{Cos}\ B) = N_T$$

that is,

$$\Sigma\left(-bE + \frac{sN}{S}\right) = N_T - \Sigma S\ \mathrm{Cos}\ B = n_T$$

The change in bearing of a line is equal to the sum of the changes in the preceding angles:

$$b_1 = a_0;\ b_2 = (a_0 + a_1);\ b_3 = (a_0 + a_1 + a_2)\ldots \text{ and so on}$$

where a_0 = change in angle at Stn 0;
a_1 = change in angle at Stn 1; and
b_1 = change in bearing of line from Stn 0; and so on.

A similar approach for the departure leads to:

$$\Sigma\left(bN + \frac{sE}{S}\right) = e_T$$

Also

$$\Sigma a = 0\ (\text{or } b_T)$$

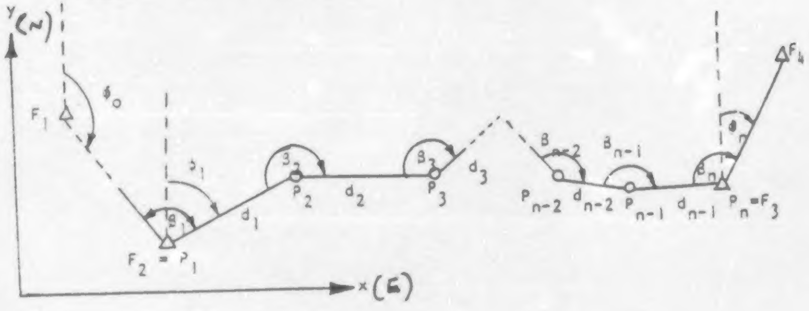
And

$$\Sigma a^2 + \Sigma\frac{1}{K^2}\left(\frac{s}{S}\right)^2 = \text{a minimum (LS condition)}$$

APPENDIX 3

NON-RIGOROUS TRAVERSE ADJUSTMENT

Traverse with Coordinate and Azimuth Control at Both Ends



The traverse starts with a known azimuth ϕ_o, determined from the coordinates of F_1 and F_2, using if necessary, the reverse geodetic problem (see Appendix 7). If all the angles β_i would be free of error, then the final azimuth $\phi_n = \phi_o + \sum_{i=1}^{n} \beta_i - n\ 180°$ would correspond to the calculated value, as determined using the coordinates of F_3 and F_4. Similarly, with accurate distance d_i, one would arrive exactly at F_3 after applying the forward geodetic problem (n-1) times.

Since this is not the case, a simple adjustment has to be performed: an "exact" least squares adjustment does not yield greatly improved results at least for small areas.

Computational Steps:

1. Calculate:

$$\Delta\beta = \phi_n - (\phi_o + \sum_{i=1}^{n} \beta_i \pm n\ \ 180°)$$

   if $\Delta\beta$ exceeds a certain magnitude a gross error has to be anticipated.

2. Correct each angle by $\frac{\Delta\beta}{n}$, since it can be generally assumed that all angles are measured with the same precision.

$$\bar{\beta}_i = \beta_i + \frac{\Delta\beta}{n}$$

3. Find the corrected azimuths:

$$\phi_i = \phi_{i-1} + \bar{\beta}_i \pm 180°$$

4. After consecutively applying the forward geodetic problem, using the measured distances and the corrected azimuths, a set of preliminary coordinates $\bar{x}_n$, $\bar{y}_n$ is obtained for P_n.

5. Find the differences in x and y:

$$\Delta x = x_n - \bar{x}_n \qquad \Delta y = y_n - \bar{y}_n$$

   If these values exceed a certain magnitude, a possible gross error has to be eliminated. This means usually additional field work

6. Distribute coordinate differences among the points at each end of any leg according to its length.

$$\overline{\Delta x}_{i/i+1} = \frac{d_i}{\sum_{k=1}^{n-1} d_k} \Delta x$$

$$\overline{\Delta y}_{i,i+1} = \frac{d_i}{\sum_{k=1}^{n-1} d_k} \Delta y$$

APPENDIX 4

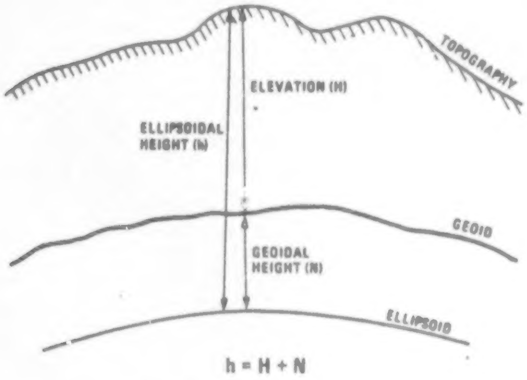
A SIMPLE TREATMENT OF HEIGHT AND ELEVATION

Two basic "heights" need to be defined: Ellipsoidal height and mean sea level elevation.

The *Ellipsoidal Height* of a point is simply the vertical distance of the point in question above the reference ellipsoid and is measured along the ellipsoidal normal from the point to the ellipsoid.

The *Elevation* of a point above mean sea level (MSL), or its orthometric height, is the vertical distance of the point above the geoid and is measured along the vertical or plumb line from the point to the geoid.

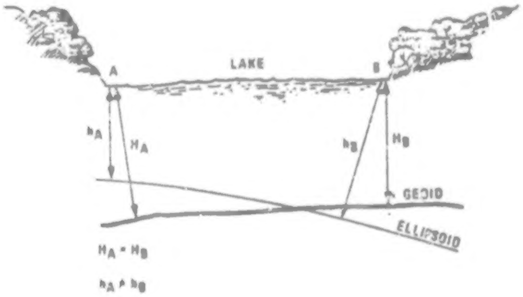
The difference between these quantities is the Geoidal Height or the vertical separation between the geoid and the ellipsoid:-



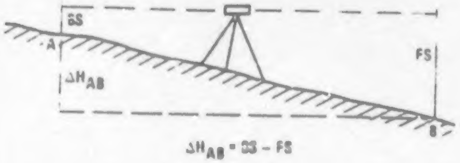
Ellipsoidal Height, Elevation and Geoidal Height

Vertical height datums of different countries and continents are referenced to MSL rather than the ellipsoid. The reason for this is two-fold. First, it is convenient to have elevations referenced to mean sea-level (the geoid) as tide-gauges may be placed around coastlines to serve as points of known level (benchmarks). Tide-gauge readings averaged over a period of several years will give "mean sea level". Second, elevations have more physical significance than ellipsoidal heights. Consider a situation where a large deviation of the vertical exists, and look at levels around the edge of a large lake.

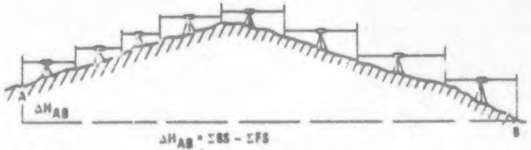
The surface of a lake will, by definition, be level as it forms an equipotential surface. Elevations will show the surface to be level (subject to restrictions discussed below), but ellipsoidal heights will generally show it to have a slope.



Ellipsoidal Height and Elevation



Spirit Leveling



A Level Run

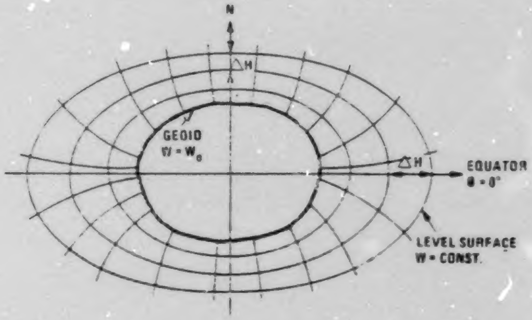
Spirit Levelling.

It is often required to combine vertical information derived from Doppler satellite positioning with existing levelling information. It is therefore worthwhile to look briefly at some of the techniques and problems of terrestrial levelling.

To measure the difference in height between two points (A and B) using spirit levelling, staffs or vertical rods are set up at each, and a level (levelling instrument) is set somewhere between them. The line of sight defined by the level is horizontal (perpendicular to the vertical) and the difference in height is found by subtracting the two staff readings. In going from A to B, the staff reading taken at A is termed a backsight (BS), while the staff reading taken at B is termed a foresight (FS)

This process may be continued over many set-ups to form a line of levels, or a level run, where the points A and B are not visible from the one level set-up.

Unfortunately, the situation is not quite this simple when A and B are separated by a considerable distance. Levelling is affected by the gravity potential. The horizontal line of sight at each level set-up is perpendicular to the vertical at that point, and hence, it is parallel to the equipotential surface at that point. As stated previously, these equipotential surfaces are usually not parallel to one another:-



The Non-Parallelism of Equipotential Surfaces

Geopotential Numbers and Dynamic Heights

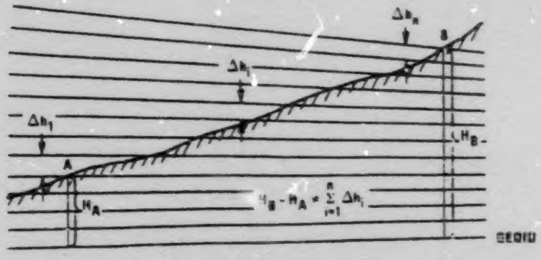
The gravitational potential (W) at a distance (D) from the earth's centre of mass may be represented approximately by:

$$W = gD$$

where "g" is the acceleration due to gravity. If "g" were constant over the surface of the earth, an equipotential surface would be equidistant from the centre of the earth. However, since "g" varies with latitude being greater at the poles than at the equator, an equipotential surface is closer to the centre of the earth at the poles than at the equator. It follows that equipotential surfaces converge as latitude increases from the equator to the poles, and that the distance between two given equipotential surfaces is less at the poles than at the equator.

The implication is that the orthometric height of a given level (equipotential) surface is dependent on latitude. (It also depends on mass distributions).

Consider a closed loop of levelling from A to B by one route and then back to A by a different route. It seems logical that the total difference in height should be zero as the starting point and finishing point are the same. However, this will not hold in practice, even if the observations are perfect, because of non-parallelism of the level, or equipotential surfaces.



The Sum of the Leveled Differences Between A and B Will Not Equal $H_B - H_A$

BEST COPY AVAILABLE

In the above figure, H_B and H_A are the orthometric heights (or MSL elevations) of points A and B. The sum of the levelled differences between A and B will not be equal to the difference in orthometric heights. There is, therefore, no direct geometrical relation between levelled height differences and orthometric height. We can, however, obtain the physical quantities of potential differences from levelled height differences if they are combined with gravity measurements. This is usually expressed as

$$W_B - W_A = -\int_A^B g\,dh$$

If the potential at A is W_a, then in relationship to the potential at the geoid, W_o, the integral is as follows:

$$W_o - W_A = \int_o^A g\,dh$$

This integral is the GEOPOTENTIAL NUMBER of A.

Geopotential Numbers are gaining recognition as a national measure of height for two major reasons:

1. All points on a level surface have the same geopotential number.

2. The geopotential number is independent of the route taken by the line of levels used to connect the point to sea level.

Geopotential numbers are measured in terms of GEOPOTENTIAL UNITS (g.p.u.) where:

1 g.p.u. = 1 kgal meter = 1000 gal meters

Hence, geopotential numbers are not in units of length, and this is the major problem in their being used as a height reference system.

If geopotential numbers are to be expressed in units of length, they must be divided by some gravity value. One type of height is DYNAMIC HEIGHT where the geopotential number is divided by the normal gravity for an arbitrary standard latitude, usually 45°.

$$H_D = \frac{C}{G}$$

where

C is the geopotential number

H_D is the dynamic height and

G is the normal gravity calculated at the standard latitude.

Orthometric heights may be derived in a similar way:-

$$H^O = \frac{C}{\bar{g}}$$

where

H^O is the orthometric height and

$\bar{g}$ is the mean value (in the integral sense) of actual gravity between the earth's surface and the geoid at the point of interest.

It has been recommended that orthometric and dynamic heights should both be used. The dynamic system is designed to be the most physically meaningful (eg. the surface of a lake will be of constant dynamic height). Orthometric heights are also important because they are directly related to ellipsoidal heights (h) by the relationship:

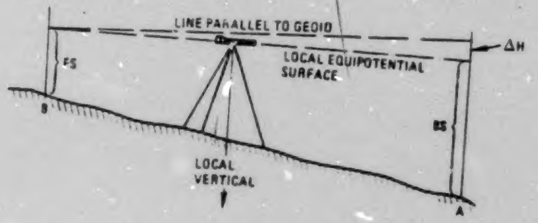
$$h = H^{O} + N$$

where N is the geoidal height above the reference ellipsoid.

Orthometric Correction

It is apparent from the foregoing that levelled height differences cannot be used directly to find differences in orthometric height. A correction, the orthometric correction, must be applied.

Consider a single set-up where a backsight and foresight are observed:-



Orthometric Correction

It can be shown that

$$\Delta H = H. \ (0.0053 \sin 2\phi)\ d\phi$$

where

H is the average elevation of the line
ϕ is the average latitude of the line
$d\phi$ is the difference in latitude of the end points, expressed in radians.

As an example, consider a line of levels, 10 km long, at a latitude of 30°, with the average elevation of the line being 1000m.

$$H = (1000)(.0053)\ (.866)\ \frac{10}{6378} = 0.0072\text{m}$$

It can be seen that this connection is proportional to north-south distance and will be very significant when very long runs of levelling such as those across continents are considered.

Although the expression given above is derived using normal gravity, it is preferable to use observed gravity where it is available. The difference may be quite significant.

Summary on Heighting

The discussion in the previous section is designed to make the user of Doppler positioning aware of the geodesist's problems in vertical control. Historically, most networks have been calculated using normal gravity rather than observed gravity. Vertical information may be presented as dynamic heights in some countries rather than orthometric heights. The difference between these is likely to be several meters; the actual difference will depend on the height of the station in question.

The vertical information given by Doppler satellite positioning is an ellipsoidal height and must be corrected for geoidal height to give orthometric height. The knowledge of geoidal height is only as good as the gravity field being used and may be in error by several meters.

The combination of all these factors means that the absolute agreement between Doppler derived and terrestrial heighting information may not be good, and it is critically dependent on a knowledge of the geoidal height. The relative agreement between the two will be much better when the unknown geoidal height error is removed by calibration at a known control station. Care must still be taken to ensure that orthometric rather than dynamic heights are being used and that any significant geoid slope is being accounted for.

APPENDIX 5

LEAST SQUARES ADJUSTMENT OF TRILATERATION AND INTERSECTION IN THE FIELD AND ANALYTICAL RESECTION IN THE FIELD

A. TRILATERATION

As mentioned in 2.1 electronic distance measurement has in recent years become both simple and accurate. As a result, many surveys which formerly would have required triangulation may now be carried out purely by trilateration, without the need for control networks whose geometrical design is optimised for base line extension. Apart from its usefulness for field coordination purposes, trilateration is particularly suitable for the periodic monitoring of critical constructions, such as dams and bridges, for movement and deformation. A minimum of two stable reference stations is required, and corrected horizontal distances, reduced to some suitable datum, are used for coordination purposes. Obviously, the use of three or four such stations is desirable, such over-determination provides redundancy and gives a better 'strength' of solution.

The rigorous adjustment of such coordination is best carried out by the least squares technique using the method of variation of coordinates. The field computation involved has been conventionally carried out by a tabular procedure. Recent developments in the versatility and memory capacity of programmable hand calculators, however, render this unnecessary; this treatment will detail a convenient, non tabular method. A program written for, and conveniently stored in, an

HP 41CV is employed. This program, by the use of controlled looping and indirect register addressing, is designed so as to minimise user participation. The surveyor is required to enter basic data only and the full least squares solution computation takes less than a minute - in the field.

The rigorous 'least squares' adjustment by the method of variation of coordinates may be briefly described as follows.

The distance between an unknown station P and a control point is given by:-

$L^2 = (\Delta E)^2 + (\Delta N)^2$ with the usual notation.

This may be linearised by differentiation.

$$2L\,dL = 2\,\Delta E . d\,(\Delta E) + 2\,\Delta N\,.\,d\,(\Delta N)$$

$$dL = \frac{\Delta E}{L} . d(\Delta E) + \frac{\Delta N}{L} . d\,(\Delta N)$$

Now

$$d(\Delta E) = dE_p$$

and

$$d(\Delta N) = dN_p$$

(because each control point is of invariant coordinates)

$$\therefore dL = \frac{\Delta E}{L} . dE_p + \frac{\Delta N}{L} . dN_p \qquad \textit{Equation 1.}$$

If approximate coordinates $E_p{}'$ and $N_p{}'$ are adopted for P, and final adjusted coordinates are denoted by E_p; N_p, then

$$N_p = N_p{}' + dN$$

$$E_p = E_p{}' + dE$$

For each distance measurement an observation equation of the form of equation 1 can be written and this may most conveniently be done in tabular format.

If a matrix notation is adopted, the result is a set of observation equations:-

$$A \times x = b + v \qquad \textit{Equation 2.}$$

where A = matrix of coefficients$(\frac{\Delta E}{L}, \frac{\Delta N}{L})$

x = vector matrix of unknowns (dE,dN)

b = vector matrix of (observed - computed) distances

v = vector matrix of adjustments to observations

If the least squares condition is applied, the solution for x is given by

$$x = (A^T WA)^{-1} A^T Wb \qquad \textit{Equation 3}$$

where W = Weight matrix.

The weighting system adopted (see Appendix 13) is decided by the surveyor: possibilities include that based on reciprocal of distance and that based on the manufacturer's specification of instrumental precision. Obviously, in cases of periodic monitoring, it is important to use the same weighting system for each successive survey, since the detection of small changes in coordinate values is the aim.

Given inputs of provisional coordinates of P, control station coordinates and reduced horizontal distances, the program computes all matrix elements for equation 2, performs the operations necessary to solve for x using equation 3, and finally outputs the final adjusted coordinates of P.

Either 3, or preferably 4, control stations may be used.

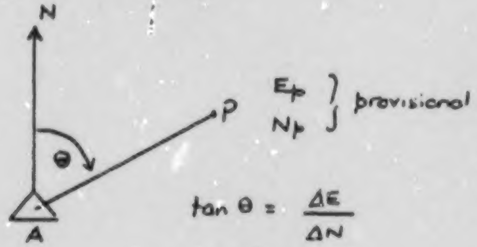
B. INTERSECTIONS

Coordination of an unknown station by bearing - bearing intersection from two control points is a common field procedure. As always, however, prudent over-determination, by the use of three or four control points (affording three or six independent solutions respectively) gives redundancy, which apart from guarding against blunders, yields a stronger final result. Although this latter may be simply achieved by meaning the separate results from all possible intersection pairs, a rigorous adjustment is not difficult to carry out, since the matrices involved are small.

As in the case of trilateration, a conventional tabular method of least squares adjustment may be replaced by a rapid and less fallible calculator procedure. Again the surveyor is required to enter basic data only, and the rigorous adjustment is done for him.

Provisional coordinates of the unknown station P are first obtained, usually by use of a simple bearing - bearing intersection program using data from two control points, or even by scaling off a map.

The coordinates of P are related in a non linear way to the observations of bearing:-



Linearising, by differentiation:-

$$\frac{d\theta}{\cos^2\theta} = \frac{d(\Delta E)}{\Delta N} - \frac{\Delta E . d(\Delta N)}{\Delta N^2}$$

Since E_A, N_A are fixed:-

$$d\theta \text{ (radians)} = \frac{\cos^2\theta}{\Delta N} .dEp - \frac{\sin^2\theta}{\Delta E} dNp$$

where dEp, dNp = coordinate adjustments to Ep,Np.
dθ = (observed - computed) term.

A similar observation equation may be written for each control station. In matrix format this may be written:-

$$A * x = b + v$$

where

A = matrix of coefficients
x = matrix of unknows dEp, dNp
b = matrix of (observed-computed) terms
v = matrix of residuals

If the least squares principle is invoked:-

$$x = (A^T WA)^{-1}.A^T Wb$$

If equal weight is given to each observation, the weight matrix,

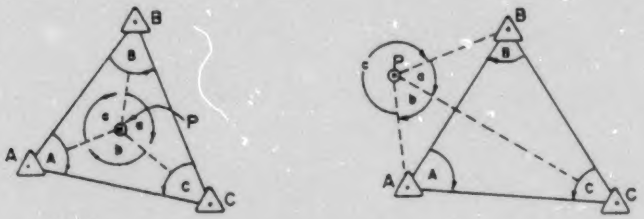
$W = I$, and then:-
$x = (A^T A)^{-1}.A^T b.$

Given inputs of provisional coordinates of P and of control station coordinates and bearings, the program computes the matrix elements, performs the operations to solve for x and then outputs the final adjusted coordinates of P. Again, the program employs controlled looping and indirect register addressing so as to minimise user participating and human error.

C. ANALYTICAL RESECTION

An alternative and often extremely convenient method of position fixing of a single point is that of resection, using three control points. If four control points are available, the redundancy allows for four independent solutions. In this case, the field surveyor might be well advised to take a mean value from these four results, if necessary applying an appropriate weighting system applicable to the particular circumstances and/or the fix geometry involved.

The Tienstra method of analytical resection is a very useful and convenient one for hand calculator programming. The unknown point P may lie within or outside the triangle formed by the three control points:-



The coordinates of P are given by

$$E_p = \frac{(n_1E_A + n_2E_B + n_3E_C)}{n_1 + n_2 + n_3}$$

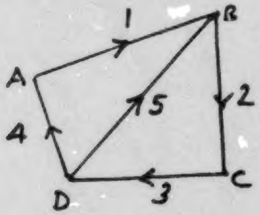
$$N_p = \frac{(n_1N_A + n_2N_B + n_3N_C)}{n_1 + n_2 + n_3}$$

where $n_1 = (\cot A - \cot a)^{-1}$
$n_2 = (\cot B - \cot b)^{-1}$
$n_3 = (\cot C - \cot c)^{-1}$

APPENDIX 6

ADJUSTMENT OF A LEVEL NETWORK BY LEAST SQUARES

Given the following observations find the most probable height of the four stations. Take the height of A = 100.000 metres



Line	Obs	Height Diff.	No. of Set Ups
AB	1 =	+ 19.612	2
BC	2 =	+ 28.054	6
CD	3 =	- 9.439	3
DA	4 =	- 38.271	6
DB	5 =	- 18.628	2

It is important to draw a diagram showing the height differences together with an arrow indicating which way the difference applies.

e.g. for line 1 we have + 19.612 and an arrow pointing from AB, which means the height of B is greater than that of A by 19.612m.

A. BY OBSERVATION EQUATIONS

As with previous problems, the first stage is to take approximate values of the desired quantities - in this case approximate height of B, C and D (A is already known). Computing height differences from A by the shortest route:

e.g.

$$B' = A + 19.612\text{ m} = 119.612\text{m}$$
$$C' = B' + 28.054\text{ m} = 147.666\text{m}$$
$$D' = A + 38.271\text{ m} = 138.271\text{m}$$

So taking each observation in turn we have:-

$$B - A = (B' + \delta B) - A = 0_1 + V_1$$
$$C - B = (C' + \delta C) - (B' + \delta B) = 0_2 + V_2$$
$$D - C = (D' + \delta D) - (C' + \delta C) = 0_3 + V_3$$
$$A - D = (A') \qquad - (D' + \delta D) = 0_4 + V_4$$
$$B - D = (B' + \delta B) - (D' + \delta D) = 0_5 + 0_5$$

Rearranying by taking known quantities over to the right we have:-

$$\delta B = \qquad 0_1 - (B' - A) + V_1$$
$$\delta C - \delta B = 0_2 - (C' - B') + V_2$$
$$\delta D - \delta C = 0_3 - (D' - C') + V_3$$
$$-\delta D = 0_4 - (A' - D') + V_4$$
$$\delta B - \delta D = 0_5 - (B' - D') + V_5$$

Substituting with approximate heights

$$\delta B = V_1$$

$$\delta C - \delta B = V_1$$

$$\delta D - \delta C = -9.439 - (-9.395) + V_3 = -0.044 + V_3$$

$$-\delta D = V_4$$

$$\delta B - \delta D = -18.628 - (-18.659) + V_5 = +0.031 + V_5$$

In matrix form

$$\begin{vmatrix} +1 & 0 & 0 \\ -1 & +1 & 0 \\ 0 & -1 & +1 \\ 0 & 0 & -1 \\ +1 & 0 & -1 \end{vmatrix} * \begin{vmatrix} \delta B \\ \delta C \\ \delta D \end{vmatrix} = \begin{vmatrix} 0.0 \\ 0.0 \\ -0.044 \\ 0.0 \\ +0.031 \end{vmatrix} + \begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{vmatrix}$$

$$A \qquad x = b + v$$

Remember we must apply weights to the observation equations.

In levelling a suitable method of weighting observations can be based on distance or sometimes number of set-ups. Often the number of set ups is proportional to the distance. However in hilly country it may be better to weight according to no. of set ups, since one requires more set-ups going up steep slopes than on level ground.

In our example we can weight proportionally to the inverse of the length or number of set ups.

i.e. $w = \frac{1}{\text{no. of set ups}}$

Our weights now become

Obs.	No. of Set Ups	Weight
1	2	0.5
2	6	0.166
3	3	0.333
4	6	0.166
5	2	0.500

From here we can form the normals and solve them in the usual way:

$$\underline{x = (A^T W.A)^{-1}.A^T W.b}.$$

B. By Condition Equations

In the example there are two loops. Circuit A B C D should equal zero. Circuit BCD should equal zero.

There are five observations m
three unknowns n
hence (m-n) conditions = 2

Thus

$$(O_1 + V_1) + (O_2 + V_2) + (O_3 + V_3) + (O_4 + V_4) = 0.000$$

$$(O_2 + V_2) + (O_3 + V_3) + (O_5 + V_5) = 0.000$$

Again we must be careful to pay attention to the signs of the lines.

In the above two circuits all the lines are going with the arrows. If we take a different circuit ABD then some of the lines now go against the arrows. So we have:

$$\vec{AB} + \overleftarrow{AD} + \vec{DA}$$

giving

$$(O_1 + V_1) - (O_5 + V_5) + (O_4 + V_4) = 0.000$$

We can insert the observations directly as listed i.e.

$$(+19.612 + V_1) - (-19.628 + V_5) + (-38.271 + V_4) = 0.000$$

Going back to the first two loops ABCD and BCD and substituting known values for observations we have:

$$V_1 + V_2 + V_3 + V_4 = +0.044$$
$$V_2 + V_3 + V_5 = +0.013$$

In matrix notation -

$$\overset{C}{\begin{vmatrix} +1 & +1 & +1 & +1 & 0 \\ 0 & +1 & +1 & 0 & +1 \end{vmatrix}} \overset{*}{*} \overset{V}{\begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{vmatrix}} \overset{=}{=} \overset{q}{\begin{vmatrix} +\ 0.044 \\ +\ 0.013 \end{vmatrix}}$$

We take the inverse weight matrix in the case of condition equations, so that the matrix uses the number of set ups.

$$C * W^{-1}\ C^T\ .\ K = q$$

After forming the normal equations = q and solving to find ka and kb we can substitude to find the values of (v):-

$$\begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{vmatrix} = \overset{W^{-1}}{\begin{vmatrix} 2 & & & & \\ & 6 & & & \\ & & 3 & & \\ & & & 6 & \\ & & & & 2 \end{vmatrix}} * \overset{C^T}{\begin{vmatrix} +1 & +0 \\ +1 & +1 \\ +1 & +1 \\ +1 & 0 \\ 0 & +1 \end{vmatrix}} * \overset{k}{\begin{vmatrix} ka \\ kb \end{vmatrix}}$$

$$\begin{vmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{vmatrix} \begin{matrix} = \\ = \\ = \\ = \\ = \end{matrix} \begin{matrix} 2.ka & \\ 6.ka & + 6\ kb \\ 3.ka & + 3\ kb \\ 6.ka & \\ & 2\ kb \end{matrix}$$

- and proceed to the solution.

APPENDIX 7

GEODETIC COMPUTATIONS ON THE SPHEROID (ELLIPSOID)

1. SYMBOLS AND BRIEF DEFINITIONS

a Major semiaxis of the reference ellipsoid (spheroid).

α Geodesic azimuth, measured clockwise from the spheroidal meridian from North through 360°.

A Normal section azimuth, measured clockwise from the spheroidal meridian from North through 360°.

b Minor semiaxis of the reference ellipsoid (spheroid)

$$b = a\ (1-f)$$

e First eccentricity of the ellipsoid (spheroid), where

$$e^2 = 2f - f^2 = (a^2 - b^2)/a^2$$

e' Second eccentricity of the ellipsoid (spheroid) where

$$e'^2 = e^2\ (1+e^2)(1-e^2) = (a^2 - b^2)/b^2$$

f Flattening of the ellipsoid (spheroid) where

$$f = \frac{a-b}{a}$$

λ Geodetic longitude measured from Greenwich, positive eastwards.

m Spheroidal meridian distance from the equator.

ν Radius of curvature of the ellipsoid (spheroid) in the prime vertical.

ρ Radius of curvature of the ellipsoid (spheroid) in the plane of the meridian.

R Mean radius of curvature;

$$R^2 = \sqrt{\rho \nu}$$

s Spheroidal distance, measured along either a normal section or a geodesic. (The difference is negligible).

ϕ Geodetic latitude, positive in Northern hemisphere.

ΔA Change in azimuth of a normal section between two points on the ellipsoid (spheroid).

$$A_{21} = A_{12} \pm 180° \pm \Delta A''$$

$\Delta\lambda$ $(\lambda_2 - \lambda_1)$

$\Delta\phi$ $(\phi_2 - \phi_1)$

2. NOTES

1. Angles and azimuths are measured in normal sections. It is seldom necessary to compute geodesic azimuths and the difference between the azimuth (α) of the geodesic and that (A) of the normal section is, for a 100 km line, equal to (0".028 $\cos^2\phi \sin 2\alpha$), which is usually negligible (0".028 when $\phi=0$, $\alpha = 45°$ even when the most accurate instruments are being used. Also, the difference between the normal section length and the geodesic length is always negligible.

2. Other symbols used are defined at appropriate points in the text.

3. SUMMARY OF PROBLEMS CONSIDERED

All of these problems have been programmed in the Department of Survey and Land Sketch and stored in an HP 41 CV hand held, battery powered programmable calculator by P. Done. They were published in THE NEW ZEALAND SURVEYOR, Nos 261 and 262.

GIVEN	COMPUTATION	
a f	b,e	Ellipsoid parameters.
a f ϕ	ν_ϕ ρ_ϕ R_ϕ	Radii of curvature
a f $\phi_1 = \phi_2$ $\Delta\lambda_{12}$ a f $\phi_1 = \phi_2$ λ_1 s_{12}	ν_ϕ ρ_ϕ R_ϕ s_{12} $\Delta\lambda$ thence λ_2	Spheroidal distance along parallel - forward & reverse problems.
a f ϕ_1	m_1 (m_{12})	Meridional are - forward.
a f ϕ m	ϕ_2	Meridional arc - Reverse.
a f ϕ_1 ϕ_2 $\Delta\lambda_{12}$	$\Delta A''$ thence A_{12} A_{21}	Meridian Convergence.
a f ϕ_1 λ_1 s_{12} A_{12}	ϕ_2 λ_2 A_{21}	"Forward" problem (polar) (GAUSS).
a f ϕ_1 λ_1 ϕ_2 λ_2 ..	s_{12} A_{12} A_{21}	"Reverse" problem (join) (GAUSS).
As above	As above	As above using rigorous rigorous formulae (ROBBINS).

4. THEORETICAL ASPECTS

Ellipsoidal (spheroidal) parameters

Given a and f, b and e are easily determined:

$$b = a - af$$

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

Radii of curvature

On the spheroid, the radius of curvature varies from point to point, unlike that of a sphere. Further, at any point, it varies with direction. At any point of latitude the radius of curvature in the plane of the meridian (ρ) is a minimum, while that in the prime vertical (ν) is a maximum: ρ and ν are described as the principal radii of curvature at any point.

For any general azimuth A, the radius of curvature R_A is given by

$$\frac{1}{R_A} = \frac{\sin^2 A}{\nu} + \frac{\cos^2 A}{\rho}$$

At any point, the geometric mean radius of curvature

$$R = \sqrt{\rho\nu}$$

This is a useful quantity because pherical formulae may be converted into spheroidal ones, with little loss of accuracy, by the substitution of $\rho\nu$ for R^2.

Specifically

$$\rho_\varphi = a \frac{(1 - e^2)}{(1 - e^2\sin^2\varphi)^{3/2}}$$

$$\nu_\varphi = \frac{a}{(1-e^2\sin^2\varphi)^{\frac{1}{2}}}$$

Spheroidal distance along a parallel

The basic formula for computing spheroidal distance along a parallel is

$$s = \frac{\Delta\lambda}{206\ 264.806} \times (\nu_1 \cos\varphi_1)$$

This is based on the fact that the radius of the parallel is given by $(\nu_1 \cos \varphi_1)$.

Two types of problem occur.

The forward problem consists of the determination of the length of arc of the parallel, given $\varphi_1 = \varphi_2$, λ_1 and λ_2.

The reverse problem consists of the determination of $\Delta\lambda''$, given spheroidal distance along the parallel, $\varphi_1 = \varphi_2$ and λ_2.

Meridional Arc

Any meridian is both a normal section and a geodesic, and the azimuths between two points on the same meridian are 0° and 180°. The spheroidal distance between two such points very close together will be given by

$$dm = \rho d\varphi$$

Integration between the limits φ_1 and φ_2 will give m_{12} :-

$$m_{12} = a\,(1-e^2)\int_{\varphi_1}^{\varphi_2}(1-e^2\,\sin^2\varphi)^{-3/2}\,d\varphi$$

This, an elliptic integral, may be evaluated using the following expansion :-

$$m_{12} = a(1-e^2)\int_{\varphi_1}^{\varphi_2}\left(1+\frac{3}{2}e^2\sin^2\varphi + \frac{1}{2}.\,\frac{3}{2}.\,\frac{5}{2}.e^4\sin^4\varphi + \ldots.\right)d\varphi$$

We may note that, since e = 1/150 for the spheriod, we may neglect terms in e where powers exceeding e^6 are involved ($e^6 \approx 0.25.10^{-9}$).

If we substitute for $\sin^2\varphi$, $\sin^4\varphi$ and $\sin^6\varphi$ terms in 2φ, 4φ and 6φ:

$$\sin^2\varphi = \frac{1}{2}\,(1-\cos 2\varphi)$$

$$\sin^4\varphi = \frac{3}{8} - \frac{1}{2}\cos 2\varphi + \frac{1}{8}\cos 4\varphi$$

$$\sin^6\varphi = \frac{5}{16} - \frac{15}{32}\cos 2\varphi + \frac{3}{16}\cos 4\varphi - \frac{1}{32}\cos 6\varphi$$

and integrate, we get, neglecting further terms.

$$m_{12} = a(1-e^2)\ A\Big[(\varphi_2 - \varphi_1) - \frac{1}{2}B(\sin 2\varphi_2 - \sin 2\varphi_1)$$
$$+ \frac{C}{4}\sin(4\varphi_2 - \sin 4\varphi_1)\Big]$$

Here,

$$A = 1+\frac{3}{4}e^2 + \frac{45}{65}e^4 + \frac{175}{256}e^6 + \ldots.$$

$$B = \frac{3}{4}e^2 + \frac{15}{16}e^4 + \frac{525}{512}e^6 + \ldots.$$

$$\frac{15}{16}e^4 + \frac{105}{256}e^6 + \ldots.$$

For the Australian National Spheroid this reduces to

$$\begin{aligned} m_{\varphi_1} &= (111\ 133.384\ 785)\ \varphi_1^{\circ} \\ \text{(metres)}\quad &- (16\ 038.954\ 6\quad)\ \sin 2\varphi_1 \\ &+ (\quad 16.833\ 1\quad)\ \sin 4\varphi_1 \\ &- (\quad 0.021\ 8\quad)\ \sin 6\varphi_1 \end{aligned}$$

Where φ_1° = latitude in decimal degrees (negative in Southern hemisphere)

It is possible however to make certain simplifying assumptions, as is done in the formulae of Gauss and Puissant, and to assume, as an implicit development of of

$$dm = \rho d\varphi$$

that $m_{12} = \rho_m\,\Delta\varphi$

where $\rho_m = \rho\left(\frac{\varphi.1' + \varphi_1}{2}\right)$

This will clearly be the more accurate, the smaller the value of $\Delta\varphi$.

Even for a line 100 km long, however, this turns out to be a reasonable assumption - the maximum error is about 1 in 4½ million (at the equator).

The direct problem therefore may be easily solved using this latter relationship - i.e. given $\Delta\varphi$, determine m_{12} . ρ_m is first determined, and this is then converted to m_{12} very easily, since $m_{12} = \dfrac{\rho m\ .\ \Delta''\varphi}{206\ 264.806}$

The reverse problem is not so straight forward. Substitution for ρ_m in the simplified formula gives

$$m_{12} = \frac{a.(1-e^2)}{\left[1-e^2 \sin^2 \frac{(\varphi_1+\varphi_2)}{2}\right]^{\frac{3}{2}}} \cdot \frac{(\varphi_2 - \varphi_1)}{206\ 264.806}$$

Given m_{12}, a, f and φ_1, there is only one unknown, φ_2, but solution of the equation for φ_2 is not a simple procedure.

Fortunately there are two convenient and rapid alternatives available (see NEW ZEALAND SURVEYOR NO. 261).

Convergence of the Meridians

The angular difference between "forward" and "reverse" azimuths A_{12} and A_{21} is not exactly 180°. The discrepancy, which cannot normally be ignored in computations on the spheroid, is due to the non-parallellism of meridians 'converging' towards the pole and is therefore termed "meridian convergence" ($\Delta A''$).

By definition

$$A_{21} = A_{12} \pm 180 \pm \Delta A$$

For any line of less than 200 km on the spheroid, which is of small flattening, meridian convergence is for practical purposes equal to that for a similar line between points of the same latitude on a sphere.

Spherical geometry, therefore, gives us

$$\Delta''A = \pm \Delta\lambda'' \sin \varphi_m \sec \frac{\Delta\varphi}{2}$$

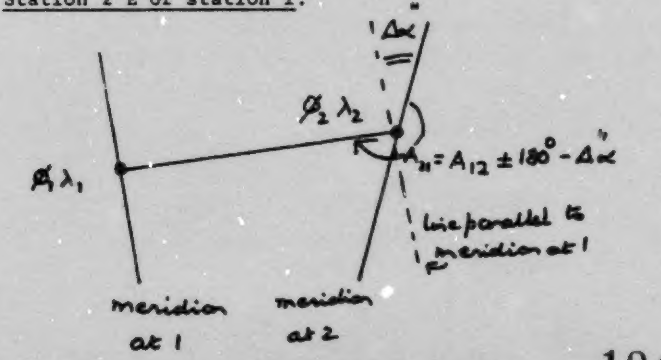
For very short lines,

$$\Delta''A = \pm \Delta\lambda'' \sin \varphi_m$$

Allocation of sign to $\Delta''A$ is left to the user: this is best done by inspection of sketch diagrams, such as the one below:-

Sphemisphere

Station 2 E of station 1.



Rules may be formulated thus:-

$$A_{21} = A_{12} \pm 180 \pm \text{convergence}$$

		Sign of convergence
N hemisphere	Station 2 E of station 1 Station 2 W of station 1	+ -
S hemisphere	Station 2 E of station 1 Station 2 W of station 1	- +

"The "forward" and "reverse" problems ("polar" and "join")

Formulae for computing on the ellipsoid give the relationships between two points 1 and 2 on the surface of the ellipsoid. Four of the seven quantities involved (φ_1, φ_2, λ_1, λ_2, A_{12} A_{21} and s) are known. The forward (or "direct", or "polar", or "radiation") problem is to determine φ_2 λ_2 and A_{21} (given φ_1, λ_1, A_{12} and s); the reverse (or "inverse", or "join") is to compute s, A_{12} and A_{21} (given φ_1, λ_1, φ_2, λ_2). Three independent equations are required; usually these are formulated in terms of expressions for $\Delta\varphi$, $\Delta\lambda$, ΔA. Those used for computations on the Australian National Spheroid are the rigorous ones given by Dr. A.R. Robbins, Puissant's formulae, and - the simplest - the Gauss mid-latitude formulae.

If terms of the 5th order are included, the Gauss mid-latitude formulae are as listed on the following page.

Gauss mid-latitude formulae (including 5th order terms)

$$\begin{aligned}\boxed{\Delta\varphi''} = \frac{s\cos A}{\rho\sin 1''}\cdot\Big[\,1 &+ \frac{s^2}{24\nu^2}\cdot\Big[2+3t^2-\cos^2 A(2+3t^2)+d^2(2-\cos^2 A(5-3t^2)) \\ &\qquad -d^4\cos^2 A(3+12t^2)\Big] \\ &+ \frac{s^4}{1920\nu^4}\cdot\Big[16+60t^2+45t^4-\cos^2 A(40+120t^2+90t^4)+\cos^4 A(24+60t^2+45t^4) \\ &+ d^2\Big(32+28t^2-\cos^2 A(156-30t^2-270t^4)+\cos^4 A(136-70t^2-270t^4)\Big)\Big]\Big]\end{aligned}$$

$$\begin{aligned}\boxed{\Delta\lambda''} = \frac{s\sin A}{\nu\cos\varphi\sin 1''}\Big[1 &+ \frac{s^2}{24\nu^2}\Big[t^2-\cos^2 A(1+t^2)-d^2\cos^2 A(1-9t^2)\Big] \\ &+ \frac{s^4}{1920\nu^4}\Big[8t^2+9t^4-\cos^2 A(8+26t^2+18t^4)+\cos^4 A(9+18t^2+9t^4) \\ &-d^2\Big(8t^2-\cos^2 A(16-270t^2-270t^4)+\cos^4 A(30-240t^2-270t^4)\Big)\Big]\end{aligned}$$

$$\begin{aligned}\boxed{\Delta A''} = \frac{st\sin A}{\nu\sin 1''}\Big[\,1 &+ \frac{s^2}{24\nu^2}\Big[2-t^2-t^2\cos^2 A+5d^4\cos^2 A+d^2\Big(2+\cos^2 A, \\ &\qquad (5+9t^2)\Big)\Big]\Big] \\ &+ \frac{s^4}{1920\nu^4}\Big[16+20t^2+9t^4-\cos^2 A(20t^2+18t^4)+9t^4\cos^4 A \\ &+ d^2\Big(32-12t^2+\cos^2 A(84+390t^2+270t^4)-\cos^4 A(100+330t^2+270t^4)\Big)\Big]\Big]\end{aligned}$$

where

$t = \tan\varphi$
$d^2 = E\cos^2\varphi$
$E = \frac{e^2}{1-e^2}$

and the arguments are

$\varphi = \frac{1}{2}(\varphi_1 + \varphi_2)$...(also used for computation of ρ, ν)

$A = \frac{1}{2}(A_1 + A_2)$

Clearly, solution of the direct problem by these formulae require iteration, using programs. Three iterations usually suffice (op.cit.) The reverse problem as already stated is not iterative is character (ibid).

The forward and reverse problems-rigorous formulae

The formulae used are those given by Dr. A.R. Robbins for the normal section. They are accurate to better than 20mm over distances of 1,500 kms. Iteration is not required.

For the forward problem:-

$$h = e' \cos \varphi_1 \cos A_{12}$$

$$g = e' \sin \varphi_1$$

$$\theta = {}^{s}/\nu_1$$

$$\sigma = \theta\left[1 + \frac{\theta^2}{6} h^2 (1-h^2) - \frac{\theta^3}{8} gh (1-2h^2) - \frac{\theta^4}{120} h^2(4-17h^2) - 3g^2(1-7h^2) + \frac{\theta^5}{48} gh\right]$$

$$\sin \psi_2 = \sin \varphi_1 \cos \sigma + \cos \varphi_1 \cos A_{12} \sin \sigma$$

$$\sin \Delta\lambda = \sin \sigma \sin A_{12} \sec \psi_2 \quad ; \lambda_2 = \lambda_1 + \Delta\lambda''$$

$$\sin A'_{21} = -\cos \varphi_1 \sin A_{12} \sec \psi_2 = -\cos \varphi_1 \sin \Delta\lambda \operatorname{cosec} \sigma$$

$$\frac{\nu_1}{r_2} = 1 + \frac{1}{2}e'^2 (\sin \psi_2 - \sin \varphi_1)^2$$

$$\tan \varphi_2 = \tan \psi_2 (1 + e'^2)\left[1 - e^2 \left(\frac{\nu_1}{r_2}\right) . \frac{\sin \varphi_1}{\sin \psi_2}\right]$$

$$A_{21} = A'_{21} - (\varphi_2 - \psi_2) \sin A'_{21} . \tan \tfrac{1}{2}\sigma$$

For the reverse problem:

$$\tan \psi_2 = (1-e^2) \tan \varphi_2 + e^2\left(\frac{\nu_1 \sin \varphi_1}{\nu_2 \cos \varphi_2}\right)$$

$$\cot A_{12} = (\cos \varphi_1 \tan \psi_2 - \sin \varphi_1 \cos \Delta\lambda).\operatorname{cosec} \Delta\lambda$$

$$\cot A'_{21} = (\sin \psi_2 \cos \Delta\lambda - \cos \psi_2 \tan \varphi_1).\operatorname{cosec} \Delta\lambda$$

$$\sin \sigma = \sin \Delta\lambda \cos \psi_2 \operatorname{cosec} A_{12} = -\sin \Delta\lambda \cos \varphi_1 \operatorname{cosec} A'_{21}$$

$$h = e' \cos \varphi_1 \cos A_{12}$$

$$g = e' \sin \varphi_1$$

$$s = \nu_1 \sigma \left[1 - \frac{\sigma^2}{6} h^2 (1-h^2) + \frac{\sigma^3}{8} gh (1-2h^2) + \frac{\sigma^4}{120}\left(h^2(4-7h^2) - 3g^2(1-7h^2)\right) - \frac{\sigma^5}{48} gh\right]$$

$$A_{21} = A'_{21} - (\varphi_2 - \psi_2) \sin A' \ .\tan \tfrac{1}{2}\sigma$$

APPENDIX 8

GEODETIC COMPUTATIONS INVOLVING SPHEROID/PROJECTION RELATIONSHIPS

1. SYMBOLS AND BRIEF DEFINITIONS

The symbols used are listed below. Many terms are also more fully defined and illustrated in diagrammatic form.

φ = Geodetic latitude, negative south of the equator.
φ_1, φ_2 = Latitude at points 1 and 2 respectively.
φ = $(\varphi_1 + \varphi_2)/2$ } and similarly for
$\Delta\varphi$ = $\varphi_2 - \varphi_1$ } other terms.
$\Delta\overset{''}{\varphi}$ = $\Delta\varphi$ expressed in seconds of arc.
λ = Geodetic longitude measured from Greenwich, positive eastwards.
$\Delta\lambda$ = $\lambda_2 - \lambda_1$
λ_0 = Geodetic longitude of a central meridian.
ω = Geodetic longitude measured from a Central Meridian, positive eastwards: $\omega = \lambda - \lambda_0$
E' = Easting measured from a central meridian, positive eastwards.
N' = Northing measured from the equator, negative southwards.
E = E' +500 000 metres.
N = N' in the northern hemisphere.
= N'+10 000 000 metres in the southern hemisphere.
ρ, ν = Radii of curvature of the spheroid in meridian and prime vertical respectively.
α = Azimuth, clockwise through 360° from true north.
β = Grid bearing, clockwise through 360° from grid north.
θ = Plane bearing, clockwise through 360° from grid north.
λ = Grid convergence, positive when grid north is west of true north, negative when grid north is east of true north. $\beta = \alpha + \lambda$
δ = Arc-to-chord correction, with sign defined by the equations: $\theta = \beta + \delta = \alpha + \lambda + \delta$

$\Delta\alpha$ = Meridian convergence.
$\Delta\beta$ = Line curvature.
s = Spheroidal distance.
S = Grid distance.
L = Plane distance.
m = Meridian distance, true distance from the equator, negative southwards.
a,b = Major and minor semi-axes of the spheroid.
e^2 = $(a^2 - b^2)/a^2$ = (eccentricity)^2
e'^2 = $(a^2 - b^2)/b^2$ = (second eccentricity)^2
k_0 = Central scale factor = 0.999 6
k = Point scale factor.
K = Line scale factor.
t = tan φ
Ψ = ν/ρ
φ' = Latitude for which m = N'/k_0
t', Ψ', ρ', ν' are functions of the latitude φ'
R^2 = $\rho\nu$
r^2 = $R^2 k_0^2 = \rho\nu k_0^2$

Note: E',N',E,N,S,L,r,k and K include the central scale factor, k_0.

s, ρ, ν, R and m are true distances, which must be specifically multiplied by k_0 when necessary.

DEFINITIONS

Azimuth, Meridian Convergence, and Spheroidal Distance

Azimuth α, is a horizontal angle measured from the spheroidal meridian clockwise from North through 360°.

Meridian Convergence $\Delta\alpha$, is the change in the azimuth of a geodesic between two points on the spheroid.

Reverse Azimuth Forward Azimuth + Meridian Convergence ± 180° ($\alpha_2 = \alpha_1 + \Delta\alpha \pm 180°$)

Spheroidal Distance s, is distance on the spheroid along either a normal section or a geodesic. The difference is usually negligible.

Grid Bearing, Line Curvature and Grid Distance

A line on the spheroid of length s is projected on the grid as an arc.

Grid Bearing β, at a point on the arc, is the angle between Grid North and the tangent to the arc at the point. It is measured from Grid North clockwise through 360°.

Line Curvature $\Delta\beta$, is the change in grid bearing between two points on the arc.

Reverse Grid Bearing = Forward Grid Bearing + Line Curvature ± 180°

$$(\beta_2 = \beta_1 + \Delta\beta \pm 180°)$$

Grid Distance S, is the length measured along the arc of the projected line whose spheroidal distance is s.

Plane Bearing and Plane Distance

A straight line can be drawn on the grid between the ends of the arc defined above.

Plane Bearing θ, is the angle between the Grid North and this straight line.

Plane Distance L, is the length of this straight line. The difference in length between the plane distance, L, and the grid distance, S, is nearly always negligible. Using plane bearings and plane distances, the formulae of plane trigonometry hold with complete rigour:

$\tan\theta = \Delta E/\Delta N$; $\Delta E = L \sin\theta$; $N = L \cos\theta$.

Grid Convergence

Grid Convergence λ, is the angular quantity to be added algebraically to an azimuth to obtain a grid bearing:

Grid Bearing = Azimuth + Grid Convergence ($\beta = \alpha + \lambda$)

In the Southern Hemisphere, grid convergence is positive for points east of a central meridian, and negative west.

Arc-to-Chord Correction

An arc-to-chord correction δ, is the angular quantity to be added algebraically to a grid bearing to obtain a plane bearing:

$$\theta = \beta + \delta = \alpha + \lambda + \delta$$

The arc-to-chord corrections differ in amount and sign at the two ends of a line. The sign is best determined from a sketch diagram. Lines which do not cross the central meridian always bow away from the central meridian. Note that ($\Delta\beta = \delta_1 - \delta_2$)

Scale Factors

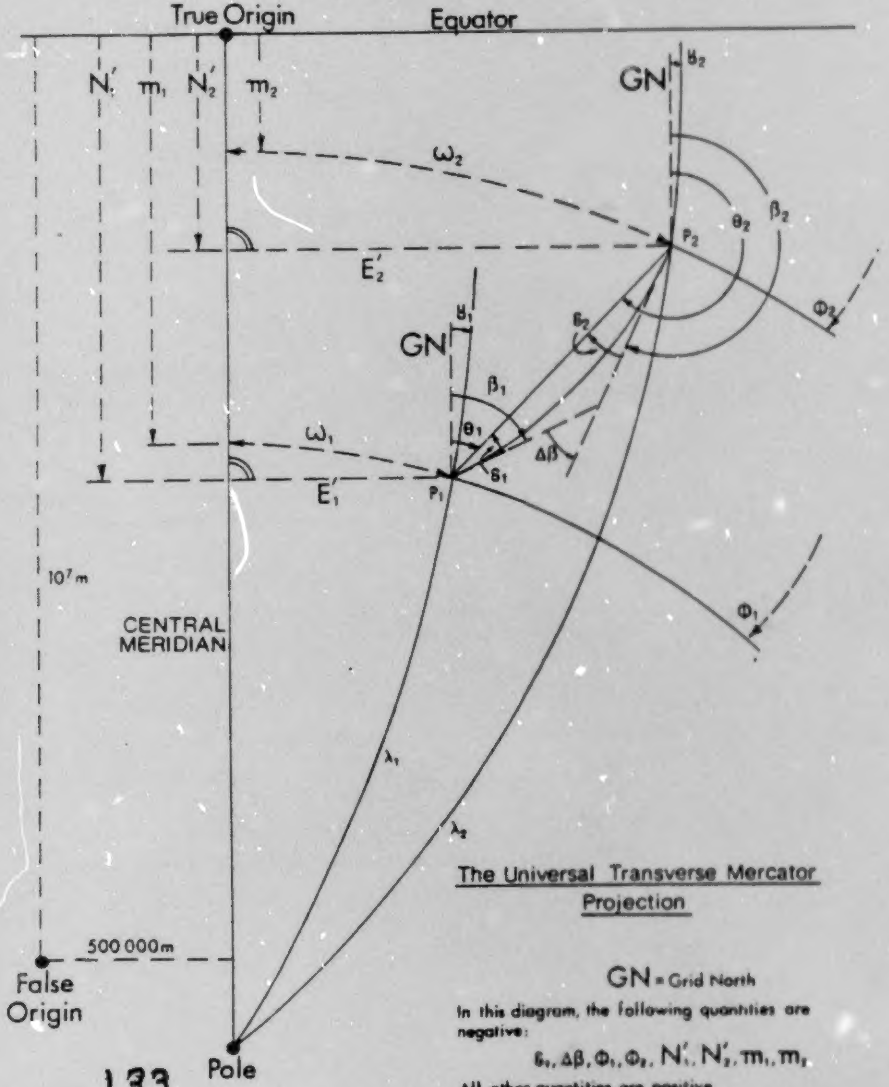
Point Scale Factor k, is the ratio of an infinitesimal distance at a point on the grid to the corresponding distance on the spheroid:

$k = dL/ds = dS/ds$

It is the distinguishing feature of conformal projections, such as the Tranverse Mercator, that this ratio is independent of the azimuth of the infinitesimal distance.

Line Scale Factor K. From point to point along a line on the grid, the point scale factor will in general vary. The line scale factor is the ratio of a plane distance, L, on the grid to the corresponding spheroidal distance, s:

$K = L/s = S/s$


True Origin
Equator
N_1'
m_1
N_2'
m_2
GN
γ_2
ω_2
θ_2
β_2
P_2
E_2'
Φ_2
GN
γ_1
δ_2
β_1
θ_1
ω_1
$\Delta\beta$
δ_1
E_1'
P_1
Φ_1
10^7m
CENTRAL MERIDIAN
λ_1
λ_2
500 000m
False Origin
Pole

The Universal Transverse Mercator Projection

GN = Grid North

In this diagram, the following quantities are negative:

$\delta_2, \Delta\beta, \Phi_1, \Phi_2, N_1', N_2', m_1, m_2$

All other quantities are positive.

2. SUMMARY OF PROBLEMS CONSIDERED

As for Appendix [illegible] all these problems have been similarly programmed, and published, by P. Done.

GIVEN	COMPUTATION
a f ϕ λ	E,N Grid convergence γ Point scale factor k
a f E N	ϕ λ Grid convergence γ Point scale factor k
a,f ϕ,E	Point scale factor k
a f ϕ_1 ϕ_2 E_1 E_2	Line scale factor K Arc to chord correction δ
a f ϕ h_m d	Geoidal distance s'

BEST COPY AVAILABLE

3. THEORETICAL ASPECTS

Orthomorphic Projection Formulae (Transverse Mercator Projection)

Redfearn's formulae are accurate to at least 1 mm in any zone of the Australian Map Grid. The basic projection formulae are (equations 1 and 2)

$$E' = k_o\Big[\nu\,\omega\,\cos\varphi + \nu\frac{\omega^3}{6}\cos^3\varphi\,(\psi - t^2) + \nu\frac{\omega^5}{120}\cos^5\varphi\Big[4\psi^3(1-6t^2)+\psi^2(1+8.t^2)-\psi(2t^2)+t^4\Big] + \nu\frac{\omega^7}{5040}\cos^7\varphi(61 - 479t^2+179t^4-t^6)\Big]$$

$$N' = k_o\Big[m+\nu\sin\varphi\frac{\omega^2}{2}\cos\varphi + \nu\sin\varphi\frac{\omega^4}{24}\cos^3\varphi(4\psi^2+\psi-t^2) + \nu\sin\varphi\frac{\omega^6}{720}\cos^5\varphi\Big(8\psi^4(11-24t^2)-28\psi^3(1-6t^2) + \psi^2(1-32t^2)-\psi(2t^2)+t^4\Big) + \nu\sin\varphi\frac{\omega^8}{40320}\cos^7\varphi(1385-3111t^2+543t^4-t^6)\Big]$$

where k_o = scale factor on the central meridian

$t = \tan\varphi$

$\psi = \nu/\rho$

m = meridian distance from the latitude of origin.

In Australia, meridian distance is defined by following terms only of an infinite series (equation 3):

$$m = a(A_o\varphi - A_2\sin 2\varphi + A_4\sin 4\varphi - A_6\sin 6\varphi)$$

where $A_o = 1 - e^2/4 - 3e^4/64 - 5e^6/256$

$A_2 = (3/8)(e^2+e^4/4+15e^6/128)$

$A_4 = (15/256)(e^4+3e^6/4)$

$A_6 = 35e^6/3072$

This limited formula is correct to less than 0.5 mm in latitude 45° anywhere within 3° of the central meridian.

On the Australian National Spheroid this reduces to (equation 4)

$$m = 111\,133.348\,785\,\varphi^\circ - 16038.9546.\sin 2\varphi + 16.833\,1.\sin 4\varphi - 0.021\,8.\sin 6\varphi \text{ metres}$$

where φ° = latitude in degrees

The reverse forms of the main equations are (equations 5 and 6)

$$\varphi = \varphi' - (t'/k_o\rho')(E'^2/2k_o\,\nu') + (t'/k_o\rho')(E'^4/24k_o^3\nu'^3)\Big[-4\psi'^2+9\psi'(1-t'^2)+12t'^2\Big] - (t'/k_o\rho')(E'^6/720k_o^5\nu'^5)\Big[8\psi'^4(11-24t'^2)-12\psi'^3(21-71t'^2) + 15\psi'^2(15-98t'^2+15t'^4)+180\psi'(5t'^2-3t'^4;+360t'^4\Big] + (t'/k_o\rho')(E'^8/4032k_o^7\nu'^7)(1385+3633t'^2+4095t'^4+1575t'^6)$$

$$\omega = \sec\varphi'(E'/k_o\nu') - \sec\varphi'(E'^3/6k_o^3\nu'^3)(\psi'+2t'^2) + \sec\varphi'(E'^5/120k_o5\,\nu'^5)\Big[-4\psi'^3(1-6t'^2) + \psi'^2(9-68t'^2)+72\psi't'^2+24t'^4\Big] - \sec\varphi'(E'^7/5040k_o^7\nu'^7)(61+662t'^2+1320t'^4+720t'^6)$$

where φ' is the latitude for which $m = N'/k_o$

φ' must be computed. All other primed quantities, t', ρ', ν', ψ' have their usual meaning but are computed using φ'.

Analysis of the contributions made by the various terms shows that the final terms of equations 1 and 2 are of the order of 10^{-5} and 10^{-7} m respectively and for practical purposes may therefore be omitted without compromising any observational accuracy. The third term of equation 2 may similarly be amended to omit all but the first term in square brackets.

The reverse equations 5 and 6 may similarly be simplified with little loss of true meaning; the last two terms of equation 5 normally contribute 10^{-4} and 10^{-7} seconds respectively. The final term in equation 6 is of the order of 10^{-5} seconds.

Grid convergence (in radians) is computed from (equation 7)

$$\begin{aligned}\lambda = & - \sin\phi\ \omega \\ & - \sin\phi\ \frac{\omega^3}{3} \cos^2\phi\ (2\psi^2 - \psi) \\ & - \sin\phi\ \frac{\omega^5}{15} \cos^4\phi \left[\psi^4(11-24t^2)-\psi^3(11-36t^2)+2\psi^2(1-7t^2)+\psi t^2\right] \\ & - \sin\phi\ \frac{\omega^7}{315} \cos^6\phi\ (17 - 26t^2 + 2t^4)\end{aligned}$$

which is the well known approximate convergence ("$\Delta\lambda\sin\phi$") plus correcting terms.
For most applications only the first two terms are needed since the next two are generally of the order of 10^{-3} and 10^{-7} seconds of arc respectively.

Point scale factor is given by (equation 8)

$$\begin{aligned}k = k_o \Big[1 & + \frac{\omega^2}{2} \cos^2\phi\ \psi \\ & + \frac{\omega^4}{24} \cos^4\phi \left(4\phi^3(1-6t^2) + \psi^2(1+24t^2)-4\psi t^2\right) \\ & + \frac{\omega^6}{720} \cos^6\phi (61-148t^2+16t^4)\end{aligned}$$

The final term here is negligible, being of the order of 10^{-10}.

Reverse forms of equations for grid convergence and point scale factor are (equation 9)

Grid Convergence (radians)

$$\begin{aligned}\gamma = & -t'(E'/k_o\nu') \\ & +t'(E'^3/3k_o^3\nu'^3)(-2\psi'^2+3\psi'+t'^2) \\ & -t'(E'^5/15k^5_o\nu'^5)\Big[\psi'^4(11-24t'^2)-3\psi'^3(8-23t'^2)+5\psi'^2 \\ & \qquad (3-14t'^2)+30\psi't'^2+3t'^4\Big] \\ & +t'(E'^7/315k_o^7\nu'^7)(17+77t'^2+105t'^4+45t'^6)\end{aligned}$$

Point Scale Factor (equation 10)

$$\begin{aligned}k = k_o\Big[& 1+E'^2/2k_c^2\rho'\nu' \\ & +(E'^4/24k_o^4\rho'^2\nu'^2)\left(4\psi'(1-6t'^2)-3(1-16t'^2)-24t'^2/\psi'\right) \\ & +(E'^6/720k_o^6\rho'^3\nu'^3)\end{aligned}$$

Again, little loss of accuracy results from some simplication. It can be shown that the third and fourth terms of equation 9 are of the order of 10^{-2} and 10^{-5} seconds of arc respectively, while the third term of equation 10 is of the order of 10^{-11}.

A line scale factor between points 1 and 2 can be computed from *either* (equation 11)

line scale factor $= \frac{1}{6}(k_1 + 4k_{mid} + k_2)$

where k_1 and k_2 are the point scale factors at the two ends of the line and k_{mid} is the point scale factor at the mid point, *or* (equation 12)

$$\text{line scale factor} = k_o\left\{1 + \left[(E'^2_1 + E'_1E'_2 + E'^2_2)/6r^2\right]\left[1 + (E'^2_1 + E'_1E'_2 + E'^2_2)/36r^2\right]\right\}$$

where $r^2 = \rho\nu k_o^2$.

The second term in square brackets can be omitted with a maximum error, on a 100 km line, of 0.25 ppm (= 25 mm) within 3° of the central meridian.

The arc-to-chord, (δ, "(t - T)" correction, in radians, from point 1 to point 2 is computed from (equation 13)

$$\delta_1 = -(N_2 - N_1)(E'_2 + 2E'_1)\left[1 - (E'_2 + 2E'_1)^2/27r^2\right]/6r^2$$

The term in square brackets can be omitted with a maximum error, on a 100 km line, of 0".08 within 3° of the central meridian.

On a Zone boundary, the arc to chord correction for a north-south line is about 0".85 per kilometre.
Equation 13 may be written as

$$\delta_1'' = -(N_2 - N_1)\,(2E'_1 + E_2 - 1.5.10^6)/6r_m^2 \sin 1''$$

and this gives results correct to 0".01 for lines 30km long.

The numerical difference between arc to chord corrections at each end of a line is very frequently neglected.
Then the formula becomes

$$\delta_1'' = -\delta_2'' = -(N_2 - N_1)(E_2 + E_1 - 10^6)/4r_m^2 \sin 1'' = \Delta\beta/2$$

which is accurate to approximately 0".6 for lines 50 km long.

Note that reduction of a reduced horizontal distance d at a mean elevation h_m to the geoid gives

$$\text{geoidal distance } s' = \frac{d.\,R\varphi}{R\varphi + h_m}$$

A program computes $R_\varphi = \sqrt{\rho\nu}$ for the mid latitude, but should that be considered unnecessary, and a typical value taken for a particular survey, this can be stored and used instead. It should be noted that, although the results are often taken as spheroidal distances (the true geoid/spheroid separation N being either unknown, or small, or both) they should strictly be corrected to the spheroid:-

$$s = s' - \frac{Ns'}{R}$$

Since the correction is approximately 1ppm per 6 m of separation, it is not negligible, and its omission can give rise to significant systematic scale errors.

APPENDIX 9

DOPPLER

The Doppler satellite system (also called TRANSIT) originally developed as a navigation aid for the United States Navy (USN) in the early 1960's. It became available to commercial users in 1967, but at that stage was used solely for navigation at sea. Further developments in technology and data-handling led to the system's use for deep water hydrographic surveying and for positioning off-shore drilling rigs. Recent years have seen the use of portable land based receivers providing results of geodetic accuracy.

The Doppler principle was discovered by Christian Doppler, an Austrian mathematician, in 1824. In experimenting with sources of wave motion, such as sound and light, he found that when a source emitting a signal at a constant frequency moved *toward* an observer, the observer received a signal which was of a *higher frequency* than the signal radiated. Conversely, if the object emitting the constant frequency moved *away* from the observer, he received a signal which was of a *lower frequency* than the actual frequency measured at its source. In short, any movement between a constant frequency measured at its source. In short, any movement between a constant frequency source and an observer produced a change in the frequency of the received signal. This effect is referred to as *'Doppler frequency shift'*. It explains the fact that a higher number of pulses are received in a given time (or higher frequency) ahead of the satellite path, and a lower number of pulses (or lower frequency) are received in the same time interval behind the satellite path.

The foundation for Satellite Geodesy was laid in 1957 shortly after the launch of SPUTNIK I. Scientists the world over noted a frequency shift in the radio signals from the satellite as it passed overhead. Then two scientists at John Hopkins University discovered by measuring this Doppler frequency shift from a station at a known location on Earth, it was possible to predict the orbital positions of the satellites. A short while later it was shown that the converse of this principle was also true; that is, if the orbital positions of the satellite are known, and the Doppler frequency shift is measured as the satellite passes, the position of stations on Earth can be mathematically determined.

Generally speaking, there are five or six navigation satellites which can be observed. Each satellite is moving in a known orbit and is transmitting on two frequencies, 400 MHz and 150 MHz. The ground station contains a 400 MHz reference frequency. As the satellite moves along its path, the signal transmitted by the satellite on the 400 MHz frequency is received at the ground station and compared with the reference frequency. The motion of the satellite toward or away from the station increases or decreases the received frequency as compared with the reference frequency. The apparent change in frequency is integrated over a given time interval producing the *Doppler count*, which translates to the change in range from the satellite's known position to the ground station unknown position. The ionospheric refraction is frequency dependent, so that a correction can be made for it by obtaining observations on two difference frequencies. For this reason, measurements are made at both 150 MHz and 400 MHz.

The TRANSIT satellites were originally designed to form a "birdcage" of evenly spaced, polar, orbits about 1075 km above the earth which they orbit about every 107 minutes. they each carry solar cells which charge the internal batteries, and signals are transmitted to the earth by an antenna, which always points downwards because of a gravity gradient stabilization boom on each satellite. As the earth rotates, it carries each point beneath each orbit in turn. Whenever a satellite passes above the horizon, a position fix is possible.

TRANSIT is operated by the Navy Astronautics Group headquartered at Point Mugu, California, with tracking stations located at Prospect Harbor, Maine; Rosemount, Minnesota; a and Wahiawa, Hawaii. Each time a TRANSIT satellite passes within line of sight of a tracking station, it receives the 150 and 400 MHz signals transmitted by the satellite, measures the Doppler frequency shift caused by the satellite's motion, and records the Doppler frequency as a function of time. The Doppler data are then sent to the Point Mugu computing centre where they are used to determine each satellite's orbit and to project each orbit many hours into the future.

The computing center forms a navigation message from the predicted orbit which is provided to the injection stations at Point Mugu and at Rosemount. At the next opportunity, one of the injection stations transmits the navigation message to the appropriate satellite. Each satellite receives a new message about every 12 hours, although the memory capacity is 16 hours.

Each TRANSIT satellite continuously broadcasts its orbital parameters (broadcast ephemeris updated every 12 hrs.). The satellite signal is modulated at precise two-minute intervals to provide a time standard. Properly equipped ground-tracking instrumentation can then record the braodcast ephemeris, Doppler counts, refraction counts, and time. Only the latter three quantities are required for the precise positioning in the Doppler System. In theory one can observe 4 passes per day at the equator and 14 at polar latitudes. (However, sometimes transmission can conflict with of those other satellites so this number is somewhat reduced in practice). A "pass" lasts from 10 to 16 minutes (during which the satellite travels 4,400 to 7,000 km) so several two minute Doppler counts may be made at each pass, the exact number depending on the altitude of the satellite. Those below 10 - 15° above the horizon are usually discarded because they are most affected by longer passages through the troposphere. The TRANSIT system is however not otherwise affected by local weather - it is an *"all-weather" system*.

A network of fixed tracking stations around the world (TRANET) monitors the satellite orbits on a continuous 24 hour basis. The combined results are computed to give the so-called *precise ephemeris*, a process which takes some time to accomplish in the USA and is only then available to certain approved users. The precise ephemeris is the term used to describe the *actual* monitored track of the satellite - in other words it is *retrospective* in character, whereas the *broadcast ephemeris* is *"predicted"*, *"on-board"*, or *"real-time"* data.

APPENDIX 10

THE GLOBAL POSITIONING SYSTEM (GPS)

The NAVSTAR Global Positioning System (GPS) is the second generation satellite navigation systme being developed to provide an observer instantaneously with his three dimensional position (to 10m), Velocity (0.1m/sec) and time continuously anywhere in the world (and in all weathers) once the system is fully deployed in 1987.

The GPS is being developed jointly by the U.S. Army, Navy and Air Force as a result of the 1973 merger of the Navy TIMATION Program and the Air Force 621B Project. Although over $400 million has been spent to date for NAVSTAR, including the launching of several prototype satellites, the total program is expected to cost the U.S. Military more than $8 billion by the end of the century. Most of this cost however will be spent on the supply to the military of approximately 25 000 user sets. As a result of the deployment of the GPS, several other navitation systems presently operated by the U.S. Government will be phased out by the early 1990's. Included among these are OMEGA, LORAN-C, and TRANSIT.

The GPS (Figure 1) is comprised of the Space Segment, The Control Segment, those components necessary to monitor and control GPS, and the User Segment consisting of all the different users who will draw on the capabilities of the system.

NAVSTAR/GPS system

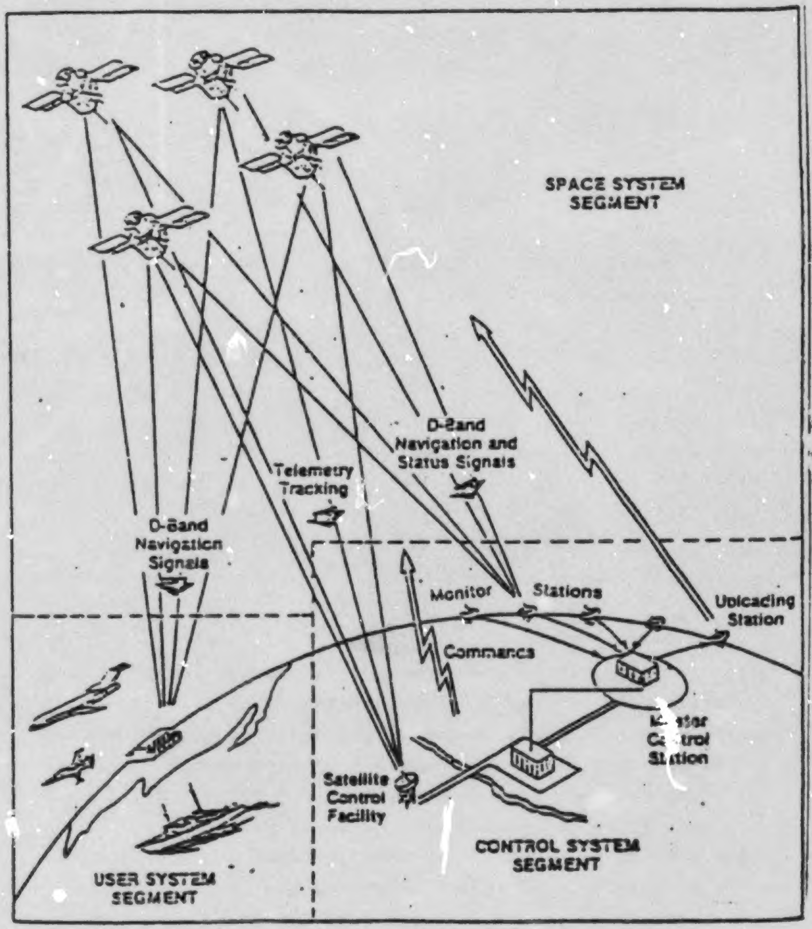

SPACE SYSTEM SEGMENT
D-Band Navigation and Status Signals
Telemetry Tracking
D-Band Navigation Signals
Monitor Stations
Uploading Station
Commands
Master Control Station
Satellite Control Facility
CONTROL SYSTEM SEGMENT
USER SYSTEM SEGMENT

Figure 1

SPACE SEGMENT

In its operational configuration (Figure 2), the GPS Satellite constellation will consist of 18 satellites in six 20 000 km circular orbits with inclinations of approximately 50°. They will be shuttle-launched from 1985 and will be deployed into six orbital planes, each plane containing three equally spaced satellites. This constellation ensures that at least five satellites will be more than 5° above the horizon anywhere on the earth, and also that on average 7 satellites may be in view, thus ensuring satellite coverage for three dimensional positioning and navigation on a worldwide basis.

The design life of the NAVSTAR Satellite is five years but each carries enough expendables to last 7 years. The "on-orbit" weight is 950 pounds and the "end-of-life" power is 400 watts. The power is supplied by two solar arrays (2.5m² each) that continually track the sun; nickel - cadmium batteries are available for eclipse operations. A 12 element shaped beam helix array provides an Earth coverage antenna pattern for the L-band navigation signals of 1575 and 1227 MHz. The signals are coherently generated and can be used to determine and correct for the effects of the ionospheric signal delay. Both navigation signals consist of a sequence of binary digits (Pseudo Random Noise) (PRN) biphase modulated on to the carrier at a rate of 10.23 megabits per second. The ephemeris and satellite clock data are modulated onto the PRN sequence at a rate of 50 bits per second. Thus, the basic navigation data is "spread" from a band width of approximately 100 Hz to a band width of approximately 20 MHz by the PRN sequence. These PRN sequences are unique to each satellite, and mutually orthogonal, permitting the use of a common carrier frequency for all satellites in the constellation. This is the P Code. The Coarse Acquisition (C/A) Signal is quadraphase modulated with the navigation signal at 1575 MHz which satisfies limited accuracy requirements and provides a rapid acquisition capability for users.

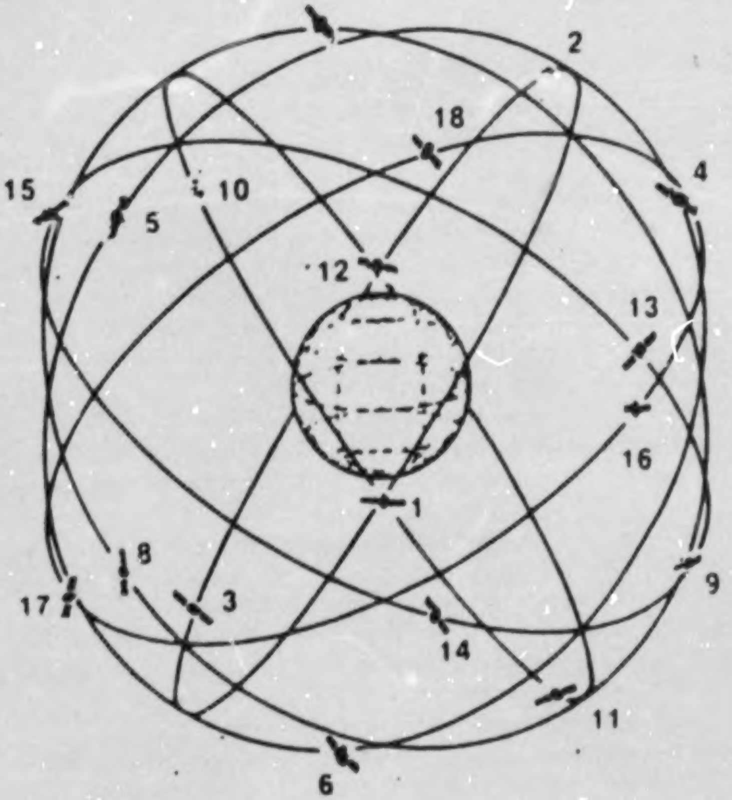


Fig 2 NAVSTAR 6-plane. 18 satellite configuration

CONTROL SEGMENT

Four ground based monitor stations located in Guam, Alaska, Hawaii and at Vandenberg Air Force Base California monitor all the NAVSTAR satellites. The data is collected at the Master control station at Vandenberg where precise estimates of orbital elements and clock offsets for each satellite are computed. This information is then uploaded at least once per day to each satellite to become part of the data broadcast by the satellite to all users. Subsequent, post event data processing will be done to generate progressively more refined information so as to define:-

(a) the gravitational field influencing the satellite motion;
(b) the location of the tracking stations;
(c) other observable system influences.

USER SEGMENT

GPS receivers take on several forms depending on the user application. The least complex configuration are C/A only receivers with single frequency capability, used for moderate accuracy, low cost, aviation or marine navigation and time transfer. At the other extreme are "full-up" PRN and C/A code receivers with dual frequency capability for high accuracy application including geodetic measurements. These receivers can have either a sequencing capability for simplicity and low cost or a multi-channel capability for simultaneous multistation reception.

The agencies developing the user equipment have studied various approaches for exploiting the GPS for geodetic purposes and have chosen the two most promising ones for continued research, development, testing, inter-comparison and evaluation. Both approaches parallel that of conventional VLBI geodesy, except the sources of the radio transmissions are the GPS Satellites instead of quasars and the collectors are portable antennas instead of massive steerable radio telescopes.

The Satellite Emission Radio Interferometric Earth Surveying (SERIES) approach further emulates conventional VLBI geodesy by treating the GPS signals as noise. The phase differencing approach requires the knowledge of the PRN code in order to reconstruct the L_1 & L_2 carrier phases.

Radio interferometry is used to measure the relative position of one point with reference to another (i.e. the vector between the two points). The radio interferometric technique measures the difference in phase of the satellite's signal at the two points occupied by the two (or more) receivers. A single phase measurement produces an ambiguous length space vector that is perpendicular to the incoming wave front. If a sufficient number of these ambiguous phase measurements are made, the unambiguous or true length of the space vector can be computed very precisely.

Because this type of receiver uses the radio interferometric technique to measure a point's co-ordinates, this instrument does not require the special coding information used in the second approach. In fact this method does not even require that the satellite orbital parameters be precisely known to determine co-ordinates accurately. An error of approximately 1 kilometer in the orbital parameters introduces an error of only 1 centimeter in the space vector. The approximate satellite parameters are required to obtain satellite predictions and to optimise the geometric configuration of the tracked satellites.

The second approach uses the ranging method using the digitally encoded signals from the satellites.

Signals are transmitted from the satellites at two L band frequencies (1227 & 1575 MHz) to permit corrections to be made for ionospheric delays in propogation times. The signals are modulated with two codes: The PRN Code which provides for the precision measurement of time and the C/A Code which provides for easy lock-on to the desired signal. Four satellites are normally required for navigation purposes and the four offering the best geometry are selected using ephemeris information transmitted by the satellites. Ranges to the four satellites are determined by scaling the signal transmit time by the speed of light. The transmitted message contains ephemeris parameters that enable the position of each satellite to be calculated at the time of transmission of the signal. Operation of the system requires precise synchronization of space vehicle clocks which is accomplished by the use of atomic frequency standards in each space vehicle, and the use of clock correction parameters that are provided by the ground Control Segment. The requirement for users to be equipped with precision clocks is eliminated by the use of range measurements from four satellites. If users maintained (very expensive) precision clocks synchronized with GPS time, positional fixes could be accomplished with only three satellites.

The measurement of range to the satellite, made by a user with an imprecise clock is called "pseudo range" because it contains a bias of fixed magnitude in each range estimate due to the clock error.

Pseudo range is defined by:

$$\bar{R}_i = R_i + C\Delta t_{A_i} + C(\Delta t_u - \Delta t_{S_i})$$

where:

$\bar{R}_i$ = pseudo-range to the satellite

R_i = true range

C = speed of light

t_{S_i} = satellite i clock offset from GPS time

t_u = user clock offset from GPS time

t_{A_i} = progagation delays and other errors

The GPS user measures the apparent (pseudo-range) transit time by measuring the phase shift of identical pseudo-range noise (PRN) codes that are generated in both the satellite and the user receiver, each synchronized with its own clock. The receiver code is shifted until a maximum correlation is achieved between the two codes. The time magnitude of the shift is the receiver's measure of the pseudo-range time.

The concept of finding the user's position based on the removal of the fixed range bias from each range estimate is illustrated in a two dimensional situation with three satellites in Figs 3 and 4. The figures illustrate that the pseudo-range radii from the three satellites do not meet at a point but enclose the shaded triangular area. However, a range value of fixed magnitude $\Delta t_u C$ can always be found that, when removed from (or added(s) the pseudo-ranges will cause the radii to meet at a point, which is the positional fix. $\Delta t_u C$ represents the range equivalent of the user's clock error.

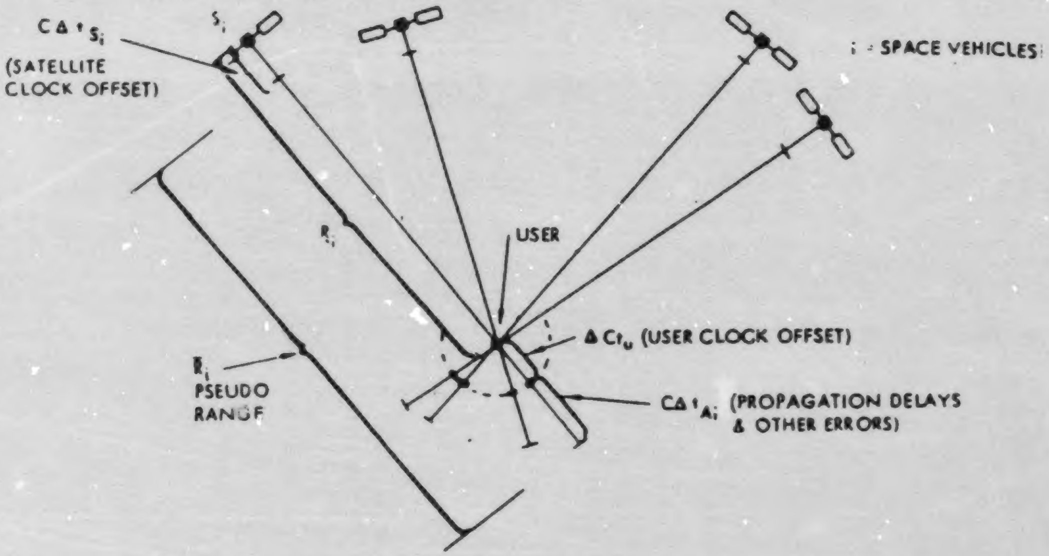

CΔt s_i
(SATELLITE CLOCK OFFSET)
S_i
i = SPACE VEHICLES
R_i
USER
ΔCt_u (USER CLOCK OFFSET)
R̄_i PSEUDO RANGE
CΔt A_i (PROPAGATION DELAYS & OTHER ERRORS)

Figure 3
Pseudo-range.

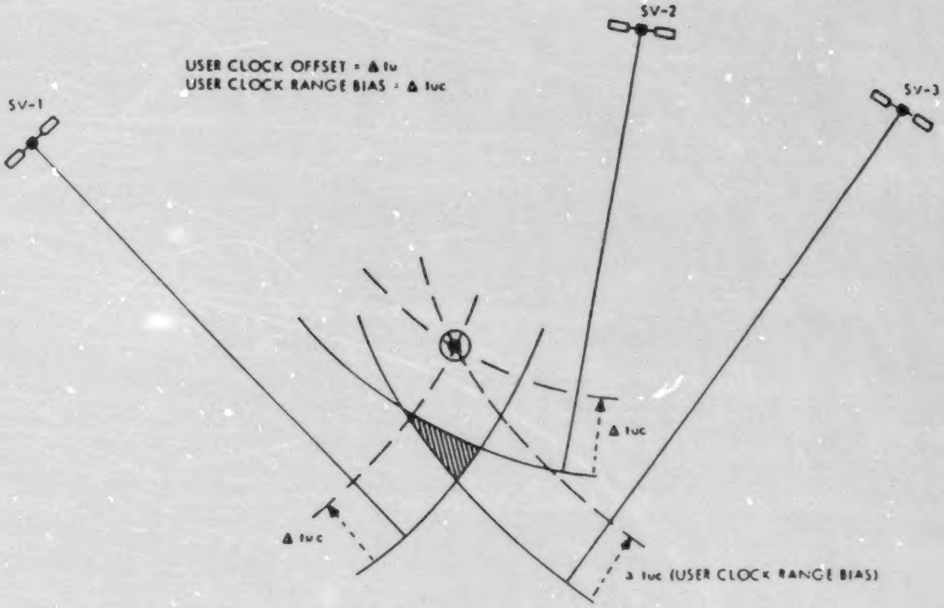

USER CLOCK OFFSET = Δ tu
USER CLOCK RANGE BIAS = Δ tuc
SV-1
SV-2
SV-3
Δ tuc
Δ tuc
Δ tuc (USER CLOCK RANGE BIAS)

Fig4—Determination of User Position and Time Offset (Two Dimensional Case).

APPENDIX 11

DATUM TRANSFORMATIONS

Since the advent of geodetic satellite positioning in the early 1970's, the subject of datums has given rise to much discussion and confusion. A datum is no more than a coordinate reference system.

The geodetic coordinates of latitude, longitude and height are measured with respect to a reference ellipsoid. This geodetic coordinate system and the associated cartesian coordinate system are illustrated in Figure 1. The size, shape, orientation and origin position of this ellipsoid define the coordinate reference system and are usually chosen so that the surface of the ellipsoid corresponds closely with the geoid in the area the datum is designed to cover. The basic reason for this choice is that when the surveyor takes observations, his instrument is measured with a spirit bubble which automatically references the observations to the level surface of the geoid. All surveying observations must be reduced to the reference ellipsoid before positions can be calculated. If the separation between the geoid and the ellipsoid is small then these reductions may not generate any significant difference and may not be necessary.

The variation in the geoid is sufficient to warrant the choice of different datums (size, shape, origin position and orientation of the ellipsoid), to achieve a good fit for different regions of the earth's surface. This shown in an exaggerated sense in Figure 2 where the North American datum is extrapolated to Europe. Even through the datum fits the geoid in the North American continent well, it does not fit the geoid in Europe. Conversely, if the European datums were extrapolated to the North American continent, the same result is found.

Before satellite Doppler positioning, nearly all surveys were carried out using monuments from previous surveys as starting points and were automatically referenced to the same datum as those previous surveys. Datums, like conventional surveys, seldom cross oceans or national boundaries and it was seldom necessary to transform the results of a conventional survey from one datum to another. But with satellite Doppler available on a world wide basis, and having sufficient accuracy to be usable in surveying and geodesy, it became necessary for datums associated with the satellite Doppler positioning system to be global in nature rather than regional. There are two satellite datums in common use. The most widely used is the "broadcast ephemeris". As the name implies this datum is defined by the operational emphemeris broadcast by each of the TRANSIT satellites. Hence, real time position computations performed by the MX 1502 will yield positions in the broadcast ephemeris datum.

The other satellite datum in use is the "precise ephemeris datum". This datum is based on satellite orbital information calculated by the Naval Surface Weapons Center using tracking data from the TRANET network. In contrast to broadcast ephemeris information which is available immediately for all of the TRANSIT satellites, precise ephemeris orbital information is not obtainable until several weeks after the survey is conducted, and is only available for a limited number of satellites (usually two). Furthermore, it is usually not made avaialble to commercial users of the TRANSIT system. For these reasons, previse ephemeris is of limited usefulness to most civilians.

Just as positions derived from conventional surveys are referenced to local or regional datums, positions derived from satellite Doppler surveys will be referenced to a satellite datum. The end goal of most satellite positioning surveys is to obtain coordinates in a local datum system. A problem arises. How does one transform the coordinates derived from satellite Doppler into coordinates referenced to the local datum?

The ideal transformation from one datum to another needs to take into account all possible differences between the two datums. The possible differences are

1. A difference in the position of the two origins. This difference is best expressed by three orthogonal shifts (dX, dY, dZ).

2. Differences in the orientations of the two ellipsoids. These differences are best expressed by rotations about the three coordinate axes (RX, RY, RZ).

3. A bias in scale between the two systems. This bias is usually expressed as a difference in parts per million (BS).

Hence the ideal transformation is one using the seven parameters, dX, dY, dZ, RX, RY, RZ, BS.

One way of obtaining these seven parameters is to solve for them in a simultaneous least squares adjustment involving points with positions determined by both satellite Doppler and conventional terrestrial techniques. The mathematical model used in the adjustment to relate the terrestrial positions, the satellite positions and the transformation parameters is usually the Bursa-Wolf model:-

$$\begin{vmatrix} X1 \\ Y1 \\ Z1 \end{vmatrix} = \begin{vmatrix} DX \\ DY \\ DZ \end{vmatrix} + (1+BS) \begin{vmatrix} 1 & -RZ & RY \\ RZ & 1 & -RX \\ -RY & RX & 1 \end{vmatrix} * \begin{vmatrix} X2 \\ Y2 \\ Z2 \end{vmatrix}$$

where (X1, Y1, Z1) and (X2, Y2, Z2) are the coordinates of a point in the local datum and satellite datum respectively. For the solution to be meaningful the points should be spread evenly over the entire region of the local datum and be sufficiently numerous to give enough redundancy to guarantee the strength of the solution. Care must also be given to the assignment of appropriate variances to both the satellite and local datum coordinates.

The data required for such an adjustment are usually only available to a national survey or mapping department and the derivation of the seven parameter set is usually best carried out by such a department. Once the seven parameters are known, they may be keyed into a MX 1502 (the Doppler receiver) which uses the Bursa-Wolf model to transform between satellite and local datum coordinates, using software incorporated into the set.

Unfortunately the seven transformation parameters are only available for a very limited number of local datums and where they are not available the surveyor is forced to use alternatives. Consider the situation where the only information known about the local datum is the size and shape of the ellipsoid. One possible procedure is to set up an MX 1502 with known coordinates in the local datum and to derive a three-parameter (DX, DY, DZ) transformation by comparison of the satellite datum point positioning result with the known local datum coordinates. This station could be the control station for a translocation survey, or multi-point network, in which case there is no extra effort involved. The only problem that arises is that precise values for the geoidal height relative to the local datum are usually not available. The local datum information is usually latitude, longitude and elevation above mean sea level while the satellite datum information is usually latitude, longitude and ellipsoidal height. The three parameter transformation (DX, DY, DZ) is calculated as a difference between the (X, Y, Z) coordinates of the point in the two datums. The (X, Y, Z) coordinates are obtained from latitude, longitude and ellipsoidal height but the ellipsoidal height in local datum cannot be obtained until the geoidal height relative to the local datum is known. This geoidal height is in turn dependent on the transformation parameters and hence a value of one must be assumed before the other can be calculated.

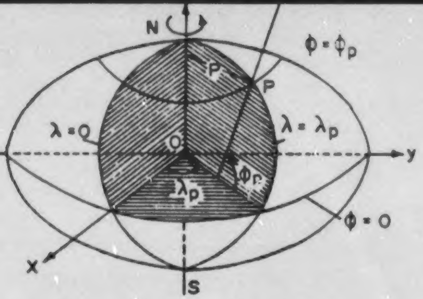
There are two possible assumptions that can be made to obtain an estimate of the local datum geoidal height to allow calculation of the three transformation parameters. First, one could assume that the local datum geoidal height is the same as the satellite datum geoidal height (see Figure 3). This is in effect assuming that the local datum ellipsoid is coincident with the satellite datum ellipsoid at the point of comparison. The second possibility is to assume that the local datum ellipsoidal height of the point is the same as the elevation above mean sea level (see Figure 4). This is in effect assuming that the local datum ellipsoid is coincident with the geoid at the point of comparison. The second assumption appears more valid in that local datums are by definition designed to fit the geoid well. The transformation obtained using this assumption should therefore be more valid than those obtained using the first assumption.

The effect of either of these assumptions is not serious if the transformation is only used to transform coordinates of points close to the point at which the transformation was derived. If the transformation is used over too large a distance these two assumptions will cause errors in the derived local coordinates. The maximum distance at which the transformation remains usable will depend on the magnitudes of the neglected rotations and scale bias (RX, RY, RZ, BS) as well as on the accuracy that is desired.

Satellite Doppler positioning surveys fall into two basic classes; those which yield an absolute or point position and those which yield a position relative to a known point (translocation). The accuracy of datum transformations is crucial to the accuracy of point positioning results as any error in the transformation parameters affects the point position directly. However, translocation results are typically insensitive to the accuracy of the transformation parameters. In fact with the MX 1502, the translocation program is designed so that it is only necessary to enter the semi-major axis and flattening of the local ellipsoid if the height of the control station above this ellipsoid is known.

In this case, a three parameter transformation is implicitly calculated and implemented using the post-translocate command. If only the elevation of the control station is known, then transformation parameters must be entered but substantial changes in the value of these parameters will only have minimal effect on the coordinates of the translocated station.

Hence, while datum transformations are a major consideration with point positioning, they are of little importance in relative positioning using translocation or network techniques. Relative positioning is increasingly becoming accepted as the standard Doppler positioning survey technique because of its relative immunity to datum transformation problems as well as its superior accuracy and convenience.


N
φ = φ_p
P'
P
λ = 0
λ = λ_p
O
y
λ_p
φ_p
φ = 0
x
S

Ellipsoidal Geographic Coordinates φ, λ.

Figure 1

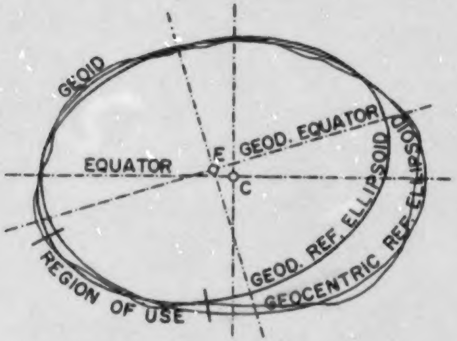

GEOID
GEOD. EQUATOR
EQUATOR
E
C
GEOD. REF. ELLIPSOID
GEOCENTRIC REF. ELLIPSOID
REGION OF USE

Figure 2

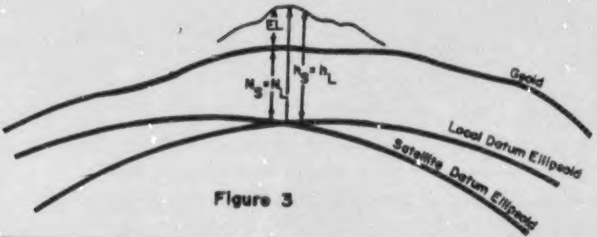

EL
N_S = N_L
h_S = h_L
Geoid
Local Datum Ellipsoid
Satellite Datum Ellipsoid

Figure 3

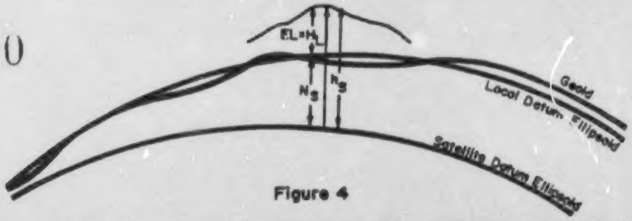

EL = h_L
N_S
h_S
Geoid
Local Datum Ellipsoid
Satellite Datum Ellipsoid

Figure 4

APPENDIX 12

GEODETIC REFERENCE SURFACES: A SUMMARY AND SIMPLE REVIEW

The physical surface of the earth is the boundary between the solid earth, or areas covered by fluid masses such as oceans and lakes and the atmosphere. In addition, knowledge of the ocean floor is now recognised as being essential in the formulation of the so-called geodetic boundary-value problem. Although the solid topographical surface is of obvious direct relevance, its irregularity renders it incapable of representation by any sort of mathematical relation. Surveyors therefore aim to delineate it by a discrete system of control points which are allocated three dimensional coordinates; as explained previously, in plane surveying (local, topographical, cadastral and most engineering surveying) the details of the land surface are adequately described in terms of a plane (horizontal) reference system, whilst geodetic surveying concerns itself with the larger scenario of national, continental and/or global control. In such fundamental work, the overall curvature of the earth must be considered. There is obvious a close relationship between global geodetic results and plane surveying, since the latter, so as to achieve some sort of consistency between surveys even at a regional or national level, should be tied to the geodetic control points. The development of national cadastres and mapping, in particular, rely on the latter.

Computations in global geodesy and geodetic surveying require compatible, "realistic" reference surfaces and systems. Since our "natural" geodetic and geophysical observations are made on the surface of the earth, most of them are influenced by the local magnitude and/or direction of the earth's external gravity field. In its irregularity, this has some analogies with those of surface topography. The geodetic problem involves therefore both geometric and gravimetric modelling. The former must aim at a geometric figure of the earth, and the latter at mathematical representation of the earth's gravity and potential fields. The latter is particularly significant for the geodetic usage of man-made satellites, and both are concerned with the "corrections" to natural observations. In fact, the geometric, gravimetric and/or geophysical aspects of the subject cannot in principle be separated.

The Geoid

Surfaces over which the earth's potential constant - equipotential, or level, surfaces- are of natural significance. By definition, at any point, the gravity vector is normal to such a surface and the local vertical direction and the horizontal plane are thereby defined. Disregarding luni-solar tidal effects, a geop is defined as an equipotential surface over which the net gravitational potential of the earth and its atmosphere is constant. The geoid may be defined as that geop which coincides, on average with mean sea level; the ocean surfaces (over 70% of the earth) closely correspond to this and it is considered to be extended under the continents so as to form a closed surface. Unfortunately, mean sea level is not strictly an equipotential due to forces such as those caused by ocean currents, variations in salinity and density, winds and pressure variations, whence the "on average" qualification above. Mean sea level itself has always historically been determined at coastal locations by long-period observations of tide-gauges (Figure 1); this in itself is a procedure fraught with both mechanical difficulties and mathematical, geophysical, oceanographical and temporal overtones. The mathematical aspects are complex and are further complicated by geophysical problems largely associated with the use of coastal sites; oceanographical effects include, inter alia, sea surface slopes and all of these may be separate from short and/or long term periodical, and secular, effects which are often unknown. Since mean sea level does not in fact form a level surface of the earth's gravity field (deviations amounting to 1 metre or more) use may be made of an isobaric surface viewed as a level surface at great depths (e.g. 1000 or 4000 m). Such techniques are referred to as oceanic or steric levelling and are techniques favoured by oceanographers.

It may be apparent from the above that although determination of mean sea level at a point may, with care and extreme attention to detail, be achieved by the use of approved standard procedures, determination of a unique equipotential surface for use as a general height datum presents a geodetic problem of great complexity even on a national basis. Further, the advent of sallite technology and in particular satellite altimetry (Figure 2) on a global scale, and the complications of sea surface topography, have been recognised as calling for a more satisfactory definition of a world vertical datum.

It is known that the geoid approximates to an oblate ellipsoid, or spheroid, and a so-called "best-fitting" theoretical ellipsoid can be orientated with respect to the geoid so that the maximum separation involved would be of the order of 100 m. The geoid itself can only be represented mathematically by an infinite series, evaluation of which necessitates truncation and therefore approximation - this again is unfortunate because of its fundamental role in surveying, so many observations being related to it. For example, in land surveying, heights are traditionally quoted as "above mean sea level" (i.e. geoid level), measured distances are often "reduced to mean sea level" for computation purposes, and all levels and theodolites are (or should be) set up with their vertical axes parallel to the local direction of gravity.

The Ellipsoid

A biaxial ellipsoid (spheroid) whose surface is an equipotential, is eminently suitable as a geodetic reference figure in preference to the geoid, to which it represents the nearest simple mathematical shape. It does not exist in the physical sense but serves as a computationally tractable model. Several such ellipsoids have been used but many of these are now of solely historical interest. Some of the more important ones are listed below.

ELLIPSOID	SEMI-MAJOR AXIS	1/f	USE
Everest (1830)	6 377 276	300.8	India and Pakistan.
Bessel (1841)	6 377 397	299.2	East Indies, Japan, N.E. China.
International (1824)	6 378 388	297.0	Europe, N. Africa
Airy (1830)	6 377 542	299.3	United Kingdom.
Krassowsky (1955)	6 378 245	298.3	U.S.S.R., E. Europe.
International (1967)	6 378 160	298.247	W. Europe.
WGS (1972)	6 378 135	298.26	Satellite geodesy.
International (1980)	6 378 137	298.26	
ANS	6 378 160	298.25	AGD-66

Particular significance attaches to the mean earth ellipsoid which represents the optimal global approximation to the geoid. In modern terms, this is earth-centred, and therefore termed geocentric; defining parameters are a, the semimajor axis, f, the flattening, w, the rotational velocity, u, the surface potential and GM, the geocentric gravitational constant. It must be noted that many ellipsoids which have proved useful for regional purposes are not geocentric, but have been so defined and so positioned in space as to provide an "ideal" local or national reference surface (Figure 3). Such local geoids do not fulfil the dual role of geometrical fit and generation of normal gravity: geocentricity, on the other hand, normally implies the assumption, or forcing, of ellipsoidal axis coincidence with the principal axes of inertia of the earth. The ANS is an example of a 'local' ellipsoid.

At any point on the earth's surface the separation N(geoidal undulation) between geoid and spheroid is in general given by the difference between the ellipsoidal height, he and the orthometric height H of the point above the geoid. Further geometrical considerations follow directly from gravimetric modelling functions associated with the ellipsoid. Theoretical equipotential surfaces known as spherops may be drawn above the ellipsoid. These continuous surfaces are everywhere convex and form a regular, oblate, spatial, layered, (onion-like) array. They are not parallel, being closer together the poles than at the equator. These theoretical equipotential surfaces of so-called normal gravity are analogous to the less regular actual equipotential surfaces of which the geoid is an example. In the case of spherops, the direction of theoretical normal gravity everywhere normal to the surface, just as actual gravity is normal the geops. The non-parallelism and curvature of the two types equipotential surface imply that in both cases the plumb line direction is not straight; in the case of normal gravity it is curved (Figure 4) in a systematic manner, but the actual plumb line curvature is more contorted and twisted due to irregularities in the local configurational geoid.

The concept of deflections of the vertical may now be considered. If to a point in the actual gravity field (on a level surface W = constant we assign a point in the normal gravity field (on a level surface U = const.) the difference between the direction of the actual plumb line and the direction of the normal plumb line is designated the deflection of the vertical. Some complications arise here because the deflection of the vertical may be defined in different ways:-

A. Definition by Helmert (Figure 5). The angle at the surface of the earth between the directions of the plumb line and the ellipsoidal normal through P. The point Q_0 on the ellipsoid is assigned to the point P as defined by the ellipsoidal normal. Knowledge of the actual or normal gravity fields between Q_0 and P is not required.

B. Definition by Pizzetti (Figure 6). The angle at the geoid between the directions of the plumb line and the ellipsoidal normal through the point P_0 on the geoid. P_0 is assigned to the surface point P by the curved plumb line. The height difference between the ellipsoid and the geoid is designated the geoid undulation N (geoid height).

C. Definition by Molodenski (Figure 6). The angle at the surface of the earth between the direction of the plumb line at P and surface normal of that spheropotential surface which has the same potential W_p as the geopotential surface through P. The points Q establish the telluroid. The height difference between the telluroid and the physical surface of the earth is known as the height anomaly ζ (ZETA). The deviation of the vertical is usually recorded as components ξ'' (XI) and 3" (ETA) in the planes of the meridian and the prime vertical respectively. Conventionally these components are reckoned as positive when the downward vertical is deviated to the south or west respectively of the downward normal. (This is equivalent to positive when the geoid is rising to the south or west relative to the ellipsoid). It can shown that:-

$$\text{XI} = \Phi - \varphi$$
$$\text{ETA} = (\Lambda - \lambda)\cos\varphi = (A - \alpha)\cot\varphi \quad (2)$$

where Φ, Λ, A are the astronomic coordinates and azimuth and φ, λ, α the geodetic coordinates and azimuth.

These Laplace equations are valid for Helmert's deflection. For the Pizzetti deflection, it is strictly necessary to allow for curvature of the normal plumb line between the actual surface point and the geoid point P_0. Due to the biaxial rotational symmetry of the ellipsoid this correction only affects the XI component and can shown to be

$$(-\ 0''.00017.\sin 2\varphi.H)$$

where H is in meters. Therefore, when computing deflection the vertical this is usually neglected, as being below the accuracy of determination of astronomic latitude, which can be affected, inter alia, by larger, local topographical effects.

$$\xi(\text{EPSILON}) = (\text{XI}\cos\alpha + \text{ETA}\sin\alpha) \quad (3)$$

is the component of the deflection of the vertical at any azimuth α. A further point of terminology stems from the fact that values of deflections of the vertical and geoid undulation depend on ellipsoidal coordinates, and consequently on both the definition the position of the reference ellipsoid. If the latter is geocentric these quantities are termed "absolute", otherwise they are "relative".

The Telluroid

This surface approximates the physical surface of the earth more closely than does the geoid. It is defined by the surface locus points such as [Q] in Figure 6, which are defined by the quantities ZETA, the height anomaly by the relationship $W_p = U_\varphi$, and by the fact that such points are at normal heights H* above the ellipsoid (Figure 7).

$$H^* = \frac{c}{\bar{\gamma}}$$

where c = geopotential number,
and $\bar{\gamma}$ = mean normal gravity.

The normal height H* of a point is equal to the height of the corresponding telluroid point [Q] above the ellipsoid. Since the telluroid itself is not a level (equipotential) surface of either actual or theoretical gravity fields it intersects spherops; it is not of course a physical reality.

The Quasigeoid

This can be considered in relation to the foregoing. If normal heights H* are projected downwards (the Helmert projection) from the earth's topographic surface, the theoretical surface so generated is termed the quasigeoid (Figure 7) the term first used by Molodenski. This in itself is neither level or real, but can best be regarded as a reference surface for normal heights H*; it is everywhere fairly close to the geoid and is, to a first approximation, coincident with it over ocean surfaces. Inland, the geoid/quasigeoid separations are of the order of metres and are strongly correlated with terrain. The quasigeoid is analogous to the geoid in that both are reference surfaces; the geoid is the reference surface for orthometric heights whereas the quasigeoid is that for normal heights. On the Helmert projection, ellipsoidal height, denoted by h, is given by

$$h = H + N = H^* + ZETA \quad (5)$$

where N is the geoid/ellipsoid separation.

It may be noted here that ZETA can in theory be computed "exactly" (Brun's formula) and that heights referred to the quasigeoid are widely used in Eastern Europe and the USSR. Despite the fact that the quasigeoid surface concept is a purely mathematical one, therefore, it is put to meaningful practical use.

The Cogeoid

Geoid determination depends on calculation of values of N with respect to the chosen ellipsoid of reference. Classically, two types of geodetic method have been applied to the problem.

The gravimetric approach was first analysed by Stokes in 1849 long before any appreciable global coverage of gravity observations was possible. In this treatment a spherical, bounding, equipotential surface is postulated - no masses being external to the geoid, and Stokes' formula gives the value of N at a point in terms of gravity anomalies over the entire earth:-

$$N = No + \frac{R}{4\pi.\bar{g}} \iint_{\sigma} \Delta g.f(\psi).d\sigma \quad (6)$$

Here:-

No = "Zero order" undulation. It is small, unknown;

and often take as zero.

R = a mean earth radius.

$\bar{g}$ = a mean value of gravity.

$f(\psi)$ = Stokes' function.

ψ = spherical distance between the point at which N is being computed and the place where the Δg values are given.

Δg = gravity anomaly, the difference between observed gravity values ("reduced" to the geoid) and the normal value of theoretical gravity at the latitude in question.

The integration is to be carried out over the complete sphere, with do an element of solid angle. The function $f(\psi)$ is effectively a factor which weights the anomalies according to their angular distances from the point considered. If any azimuth/angular system of coordinates is used, equation 6 may be written as

$$N = \frac{R}{4\pi\bar{g}} \iint \Delta g .\ f(\psi).\ \sin \psi . d\psi \quad (7)$$

The weighting function is now ($f(\psi) \sin \psi$); its relatively slow convergence to zero (Figure 8) has been a major source of geodetic exasperation ever since Stokes first published his work in 1849. It implies that the calculation of N at any point is in fact dependent on a knowledge of Δg over the whole earth. Even today there are barely sufficient data world-wide to make an accurate calculation anywhere, gaps being particularly evident over the oceans and in the southern hemisphere generally. The method of computing N has involved the process of numerical evaluation of contributions made for example by 1° x 1° blocks on a world-wide basis, first done by Hirvonen in 1934.

It should be noted that there are a number of problems relating to the application of Stokes' equation, and many of these are related to the precise nature of the gravity reductions - observed values have to be "reduced" to the geoid by taking into account elevations, and the densities of the intervening masses, and of course, use of a consistent world-wide vertical datum is implicit.

The values for Δg can thus be defined, and obtained, in various ways and in all of these some approximation is inevitable. As a result the values for N will differ for each method employed - each giving a so-called cogeoid, there being as many cogeoids as there are methods of "incorporating" the mass external to the geoid into the earth.

In contradistinction to the world wide gravimetric approach outlined above, the astrogeodetic technique is particularly suited only to the determination of values of N for land areas. The astrogeodetic determination of the geoid employs deflections of the vertical as defined by equations 1, 2 and 3; the shape of the geoid can then be determined thus:- along a line AB length l and azimuth α, ε(equation 3) is the angle between the geoid and the ellipsoid. If this were constant along the line:-

$$\Delta N = (N_B - N_A = \varepsilon . l \quad (8)$$

Since in general ε varies with l, $\Delta N = \int_A^B \varepsilon . dl$ (9), but the variation of ε with l is not known. Points A and B are therefore taken to be reasonably close and a mean value for ε is adopted:-

$$N = (\varepsilon_A + \varepsilon_B).l.\ \sin 1'' \quad (10)$$

where $\varepsilon_A = (Xl_A \cos \alpha + ETA_A \sin \alpha)$ (3)

and $\varepsilon_B = (Xl_B \cos \alpha + ETA_B \sin \alpha)$ (3)

Thus the essential procedure is to take astronomic observations at two closely spaced (the distance depends on topography and should always be below 30 km) stations, whose geodetic coordinates are obtained from computation on the particular reference ellipsoid. The Laplace equations 1 and 2, used in conjunction with 10, lead to a value of N: if N at one station is either known or abitrarily defined then its value at the other is directly obtained. In practice this procedure can be carried out so as to produce loops (so that they may be adjusted) or along lines of constant azimuth (geoid profiles). Where astronomical data are sparse and long distances are involved between A and B (100 - 200 kms are not uncommon) some form of intepolation is required. For this, astrogravimetric methods are employed, using additional and comparatively easily acquired gravimetric observations. The Vening Meinesz' integrals:-

$$\left.\begin{matrix} XI' \\ ETA' \end{matrix}\right\} = \frac{1}{4\pi\bar{g}} \cdot \int_{\alpha=0}^{2\pi}\int_{\psi=0}^{\pi} \frac{\Delta g . \partial f(\psi) \sin\psi}{\partial\psi} \left\{\begin{matrix}\cos \alpha \\ \sin \alpha\end{matrix}\right\} d\psi . d\alpha \qquad (11,12)$$

enable computation of the deflections of the vertical to be computed numerically by zone, block or template techniques. These functions fortunately converge much more rapidly than the Stokes' integral and so the effect of outer zones can be neglected, although a good coverage of gravity observations in the vicinity of the point considered is essential. XI' and ETA' so determined are generally different from the XI and ETA values determined astronomically at each point. However the discrepancy should be relatively small; in any case it varies only slightly with distance so that linear interpolations can be carried out.

It may be noted that both the Stokes' equation for his proposal for the gravimetric technique and the Vening Meinesz' equation for the astrogeodetic method both employ the gravity vector in either magnitude or direction, and obtain Δg values by comparison with theoretical normal gravity. Of these two general approaches, the gravimetric one is in principle the only capable of world-wide application, since accurate astronomical data have only so far been acquired on land. In addition, it yields absolute values for N, the reference ellipsoid being geocentric. However the astrogeodetic method is necessary to provide orientation (longitude) and scale, so that the methods are ideally combined. They can now be supplemented by additional information supplied by the use of satellite data.

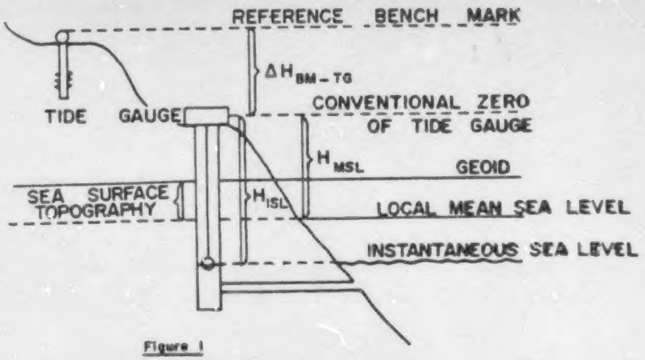
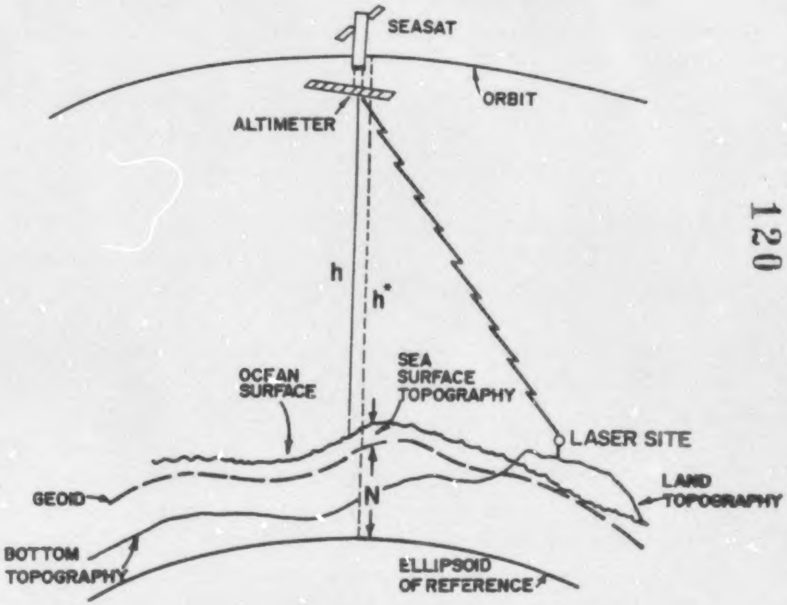

REFERENCE BENCH MARK
ΔH_{BM-TG}
TIDE GAUGE
CONVENTIONAL ZERO OF TIDE GAUGE
H_{MSL}
GEOID
SEA SURFACE TOPOGRAPHY
H_{ISL}
LOCAL MEAN SEA LEVEL
INSTANTANEOUS SEA LEVEL

Figure 1


SEASAT
ORBIT
ALTIMETER
h
h*
OCEAN SURFACE
SEA SURFACE TOPOGRAPHY
LASER SITE
LAND TOPOGRAPHY
GEOID
N
BOTTOM TOPOGRAPHY
ELLIPSOID OF REFERENCE

SCHEMATIC REPRESENTATION OF SEASAT ALTIMETRY

N = geoid / ellipsoid seperation;

h* = instantaneous height of orbit above ellipsoid;

h = instantaneous altimeter value.

Figure 2

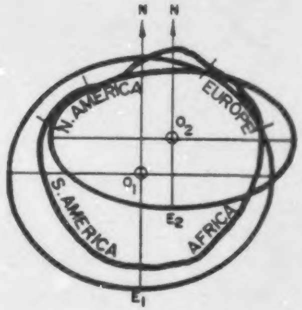
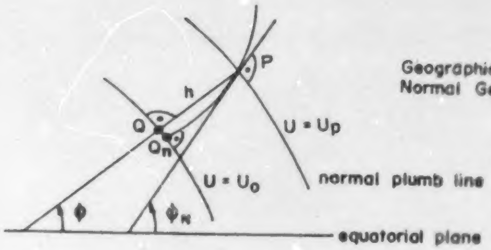
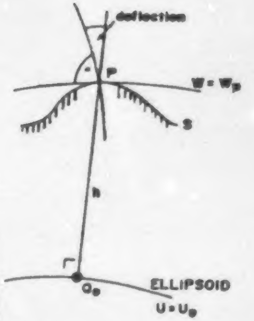

N
N
N. AMERICA
EUROPE
O_2
O_1
S. AMERICA
AFRICA
E_2
E_1

Figure 3


P
h
Q
Q_N
$U = U_P$
$U = U_0$
normal plumb line
ϕ
ϕ_N
equatorial plane

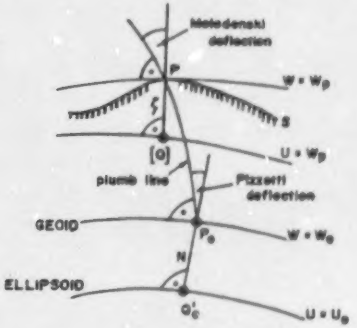
Geographic Latitude ϕ and Normal Geographic Latitude ϕ_N

Figure 4


deflection
P
$W = W_P$
S
h
Q_0
ELLIPSOID
$U = U_0$

Surface Deflection of the Vertical after Helmert.

Figure 5


Molodenski deflection
P
$W = W_P$
ζ
S
[Q]
$U = W_P$
plumb line
Pizzetti deflection
GEOID
P_0
$W = W_0$
N
ELLIPSOID
Q_0'
$U = U_0$

Deflection of the Vertical after Molodenski and Pizzetti, Height Anomaly ζ, Geoid Height N.

Figure 6

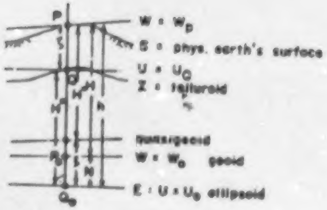

P
$W = W_P$
S = phys. earth's surface
$U = U_Q$
Σ = telluroid
Q
H
h
quasigeoid
$W = W_0$ geoid
P_0
ζ
N
$E : U = U_0$ ellipsoid
Q_0

Figure 7

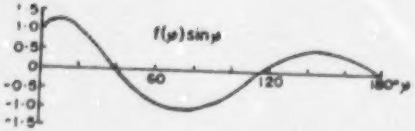


Figure 8

APPENDIX 13

SOME ELEMENTARY NOTES ON WEIGHTING

Two general aims in survey computations are the determination of the most probable values (MPV's) of observed or derived quantities and the achievement of consistency in any scheme, network or system involving such quantities. The measurements involved may be, due to a variety of reasons, of different quality or reliability and as a result it is necessary to take this into account in computing MPV's. Similarly, over-determination, necessarily involving redundant observations and therefore adjustment, is standard practice, and survey adjustment procedures are aimed at achieving both the above aims. In so doing the variability of observational quality should be taken into account - i.e. "weighting" systems should be used. It is the objective of this brief Appendix to review in an elementary way the various methods of weighting observations in a range of common survey operations.

DEFINITIONS AND GENERAL CONSIDERATIONS

An observation is one made directly on the quantity being determined, e.g. the single measurement of an angle, or an EDM distance.

As Indirect Measurement is one in which the observed value is derived from the measurement of some related quantities, e.g. the sides of a triangulation, the measurement of an angle by repetition (a multiple of the angle being measured).

A Conditioned Quantity, as opposed to an independent quantity, is one whose value must bear a fixed relationship to some other quantity or quantities. Most commonly, such relationships are dictated by geometrical considerations.

The Most Probable Value of a quantity is that more likely than any other to be its true value, and is the best result which can be attained.

A True Error is the difference between the true and observed values of a quantity. Since the true value of a quantity cannot be ascertained, the true error is never known. In case, however, of the angles of triangles or polygons, the true error of their sum is known.

A Residual Error or Residual is the difference between an observed value and the most probable value of the quantity.

The Weight of an observation is a measure of its relative trustworthiness or reliability.

In the course of a series of observations, the personal, instrumental, or atmospheric conditions may vary, so that a uniform degree of reliability in the results may not be obtained. The results are then said to be of unequal weight. Numercial values assigned as the weights of a series of observations are simply ratios indicating the relative reliabilities of the observations. Weights are assigned by estimation, or in terms of the number of separate observations of equal reliability from which the result being weighted is derived. They may be pure numbers (dimensionless) or have dimensions, as indicated below.

It can be shown theoretically that the weight (w) of an observation is inversely proportional to the square of σ, its standard error (or standard deviation):-

$$W = \frac{}{\sigma^2} \quad \text{i.e. } W = (\text{variance})^{-1}$$

In this case, w has dimensions (e.g. length $^{-2}$) and this is still true if all the observations in a particular problem have their weights multiplied by a constant k:

$$w = \frac{k}{\sigma^2}$$

It may be emphasised that although k may be introduced for computational convenience, the final result should be the same as would be obtained were k put as 1; it is the *relative* values of weights which are of direct importance.

Weights may be pure numbers (i.e. dimensionless) in all cases where observations have been made with the same precision (e.g. sets of observations by the same observer, using the same instrument in general circumstances and conditions considered to be the same). In such cases the MPV would be the weighted mean of the sets of observations, with each mean value m_i being alloted a weight equal to the number of observations n_i in each set:

$$MPV = \frac{[n_1 m_1 + n_2 m_2 + n_3 m_3 + \ldots\ldots + n_N m_N]}{[n_1 + n_2 + n_3 + \ldots\ldots + n_N]}$$

This should be considered to apply to all the succeeding sections as appropriate.

It is obviously more difficult to adopt such a clear-cut, objective approach when combining observations with different degrees of precision. In theory the (variance)$^{-1}$ relationship is straightforward, but there is a considerable body of opinion which regards weighting according to reciprocal of variance as being too optimistic. The reasoning behind this is that simply repeating a measurement does not necessarily involve resampling all the sources of random error. Possible sources of error when reading a direction

from a theodolite, for example, may involve (possibly inter alia) the following sources of error:-

(a) incorrect bisection of the mark;

(b) incorrect reading of the scale;

(c) incorrect centring of the instrument;

(d) incorrect levelling of the theodolite;

(e) lateral refraction.

At the same theodolite set-up, (c) (d) and (e) would in effect be *systematic errors* for the particular station, although they would themselves vary in a *random manner from station to station*. Only (a) and (b) of the above would contribute to the variance at the station itself and this would result, as a result of the computed variance being too small, in too optimistic a measure of the precision of the result, and therefore in too large a weight. There is probably no definitive answer to this problem because it is in general impossible to know the 'correct' variances and especially the covariances of survey observations.

Although, therefore, in theory, weighting should devolve upon the knowledge of 'reliable' variances (and covariances where applicable), it is acknowledged that, in practice, weighting schemes often lend themselves to a considerable degree of subjectivity: the latter can often, it is claimed, when based on a good knowledge of instrument, observer and conditions, together with 'a posteriori' analysis of previous such exercises, or external comparisons, such as triangular misclosure, lead to results of adequate precision and/or accuracy. In addition, empirical or semi-empirical weighting schemes have found relatively widespread use throughout the range of land survey operations. The following paragraphs will attempt to summarise those of most importance to the field surveyor.

HEIGHTING

Random errors in spirit levelling are usually assumed to be proportional to the square root of the length of the levelling line, so the weights of height differences from individual lines are taken to be inversely proportional to the lengths of the line. However, even in precise levelling, systematic errors are known to exist, due, for example, to:-

(a) steadily decreasing refraction during the day;

(b) systematic sinking or rising of the staff or instrument;

(c) small, or hysteresis, malfunctions, in the compensation mechanism of automatic levels;

(d) height differences - especially unequal refraction between foresight and backsight when levelling uphill/downhill (the worst effects being over long easy grades such as railway lines);

(e) tidal effects (the 'solid' earth is affected by these);

(f) latitude differences - especially on N/S lines, causing obliquity of lighting on staff faces.

An overall quantification could be stated as follows -
For random errors only: $J = n.\sqrt{L}$
where L = distance in km
n = random error (mm/km$^{\frac{1}{2}}$).
This suffices for normal misclose specifications.

In more detail, since systematic error is present:

$$\sigma = (n^2 . L + m . L)^{\frac{1}{2}}$$

where m = systematic error (mm/km$^{\frac{1}{2}}$)

Since the systematic error itself changes, an effective summary could be:-

1 - 5 km	(SHORT DISTANCES)	$\sigma = n.\sqrt{L}$
$5 < L < Z$	(MIDDLE DISTANCES)	$\sigma = p.\sqrt{L}$
$L > Z$	(LONG DISTANCES)	$\sigma = q.\sqrt{L}$

(The systematic error itself becomes random eventually). Here, p increases from n to q as L increases to Z; the values of D, n, p, and q may be specified locally.

For normal engineering and local surveys it is well known that this "Rolls Royce" approach is ignored.

For short levelling runs in difficult, hilly terrain it is often prudent to allot weights according to the reciprocal of the number of set ups.

Trigonometrical heighting should be carried out by reciprocal measurements - little or no weight can be given to lines observed in one direction only.

It is not uncommon to weight reciprocally observed height observations as L^{-2}, vis-à-vis spirit levelled results weighted as L^{-1}.

EDM DISTANCE MEASUREMENTS

Standard errors of distances are difficult to estimate since there is not the same simple geometrical check which exists for angles, and distances have somehow to be related to the basic international definition of the metre. This presents a scale problem which can lead to misconceptions for example regarding the accuracy of some geodetic networks.

It is common field practice with precise modern equipment to take a relatively small number of observaitons of slope distance, often less than 10, when traversing or using trilateration techniques. Ideally these should be taken in forward and reverse directions - this involves little extra effort, since, in any case, forward and reverse zenith angles are required for reduction purposes and both stations have to be occupied. The weight accorded to each mean value is most commonly L^{-1}, but some alternative approaches to the trilateration adjustment problem exist, which take the "angle of cut" into account. If, and this is now relatively unusual, a larger number of observations is taken (in excess of 30) then weighting according to the reciprocal of the variance achieved may be used. Following reasoning similar to that given above, this is still likely to be over-optimistic.

An obvious approach is, perhaps, not to use the above so-called "internal evidence" but to rely on the manufacturer's specification of precision, usually quoted in the form (x mm ± y ppm). For example, in the Geodetic Model of Australia (GMA 1980) the variance of geodimeter distances was $(1 \text{ cm} + 1 \text{ ppm})^2$ whereas that of the Tellurometer MRA1 distances was $(5 \text{ cm} + 5 \text{ ppm})^2$.

Clearly the best solution is to employ an instrument which has been calibrated over a purpose-built base which has itself been properly calibrated, ideally with a laser interferometer. On such base lines measurements can be made over a variety of distances related to the instrument's modulated wave length and a reliable "a posteriori" estimate of its performance computed to yield the standard error of a single observation.

ANGLES OR DIRECTIONS

If a number of rounds have been made, mean and σ^2 may be computed in the usual way: as explained above, σ^2 will in practice always be too small, and the weight $w = (\sigma^2)^{-1}$ too optimistic.

Assuming that in a given scheme the same quality of instrumentation, observers and techniques apply, i.e. that all errors are from the same "population", then Ferrero's formula for a triangular misclosure can be used. If Δ is the triangular misclosure, for n independent triangles,

$$\sigma^2 \text{ (angle)} = \frac{\Sigma\Delta^2}{3n} \quad \text{(as there are 3 angles in a triangle)}$$

$$\text{and } \sigma^2 \text{ (direction)} = \frac{\Sigma\Delta^2}{6n} \quad \text{(as there are 6 directions in a triangle).}$$

This is a very powerful method; it has the major advantage of being able to check sums of angles, or directions, against a known correct answer, which is a very rare situation in land surveying and measurement science generally.

It is important that the method only be used if proper rejection criteria have been used at the time of observation, otherwise [illegible] will be too small.

MATRICES FOR LEAST SQUARES TRAVERSE ADJUSTMENTS

When least squares methods are used in adjustments of precise traverses by condition equations, provisional co-ordinates are adopted, these being in any case necessary for computation of the scale factors and arc-to-chord corrections required for such high order work. The adjusted values must fulfil three conditions - closures in bearings, eastings and northings. The equations for formulated may be expressed, in matrix form, by

$$C.v = q$$

Where q is a vector matrix of misclosures
v is a vector matrix of residuals
(adjustments to observations)
and C is a matrix of coefficients.

The least squares solution is obtained by minimising $(v^T.W.v)$, which is the same as minimising $(v^T.W.v - 2\,(C.v-q)^T.k)$ where W is the weight matrix and 'k' the vector matrix of so-called 'correlates'. The latter is an intermediate result obtained from

$$K = (C.W.^{-1}C^T)^{-1}q$$

and $v = W.^{-1}C^Tk.$ (See Appendix 2)

Here the weight matrix W must take into account both linear and angular measurements, quantities obviously measured in different units. If, for the end of a leg, a constant

$$K = (\text{linear error ratio})/(\text{angular error ratio})$$

is defined the value of K should be determined for each line so as to allow for the different error ratios, and hence weight.

For a particular line of length L this can be written as

$$K = \frac{(x + L.y.\text{ppm})}{L} + (\text{angular error ratio})$$

where x = zero error
y = fractional error.

(c.f. manufacturer's specification of "x mm ± y ppm" quoted above).

The angular error at a station is probably best estimated from past experience of similar traverses and the angular standard errors found in them, in which case

$$K = \frac{x + L\,y\,\text{ppm}}{L} + \frac{L\sin\alpha}{L}$$

where $K = (x + L\,y\,\text{ppm})/(L.\alpha''.\sin 1'')$

Thus K is a measure of the relative effect of linear and angular errors. If weight is taken to be $(\sigma^2)^{-1}$ then in the least squares condition equation, elements representing the linear residual errors should be weighted by $(K^2)^{-1}$ whilst those representing the angular residual errors should be weighted by unity.

It must be noted that not only is this just one possible solution to the problem, but that in theory the weight matrix is square, and therefore involves off-diagonal terms, which express correlations between errors in the quantities measured, and which are termed covariances. It is very rare in practice for it to be possible to estimate covariances and it is usual to ignore all correlation, and to assume W to be a diagonal matrix containing no off-diagonal terms. (Often this is done even when correlation is known to exist, simply because it is so difficult to estimate).

FINAL REMARKS

If the weights are "incorrectly" chosen, does it matter? What will be the effect on the final results, or coordinates? For any individual computation on network this can be determined simply by carrying out several adjustments with different (but all reasonably logical) weights. It is impossible in fact to give a general guide, since in this field there is no such thing as "absolute truth", and in addition, networks which are of different sizes, or of different proportions in terms of angles, distances and azimuths almost certainly react differently to application of different weighting systems.

APPENDIX 14

INERTIAL SURVEYING SYSTEM (I.S.S)

Surveying systems based on the principle of inertial navigation are relatively new in geodetic practice. The development of these systems in the early seventies was sponsored by military agencies, in particular, the US Army Engineer Topographic Laboratories at Fort Belvoir, Virginia.

In 1975, the first systems of this kind became available to nonmilitary users. These systems were tested extensively during that year and, since 1976, they have been used for production work by government agencies and private industry in Canada and the United States. The bulk of the work to date has been in establishing geodetic control that generally has second-order accuracy. Recent results indicate, however, that it may be possible to achieve first-order accuracy.

The application to cadastral surveys will not be discussed here, but some of the following points may to help to illustrate the use of the system. The current use of the system for control surveys and densification indicates one large area where inertial survey systems can be used to advantage. Favorable aspects of the systems include the speed of the survey, independence of lines of sight, homogeneity, output in coordinate form and, last but not least, the lower costs of the survey. In cadastral surveys, an obvious sequence would consist of Doppler control of 100 km to 300 km apart, inertial densification to 2 km to 10 km, and then a detailed survey. At present, there are no data available as to the usefulness of the system in a detailed cadastral survey with a large number of points only a short distance apart. The obvious procedure here would be the accurate positioning of traverse poitns by the system and the measurement of all neighboring points in a tacheometer fashion at the same time.

The azimuth would come from the system, the distance from attached EDM equipment. One of the systems now offers the EDM equipment as an optional feature. The system may also be of interest for integrated data banks because it determines horizontal position and height, together with deflections of the vertical and gravity anomalies.

In classical mechanics, an inertial coordinate system is one in which the Newtonian equations of motion hold. It can be visualized as a system nonrotating with respect to the distant galaxies. This classical approach has been chosen because it is more intuitive and, therefore, better suited for an introductory exposition, even though it may be too simple for mathematicians.
We begin with Newton's second law of motion in the special form

$$f = m\,a$$

where the mass m is considered to be constant. With f measured and m known, the acceleration a can be computed. Using a Cartesian, coordinate system, the vector of accelerations a can be expressed as second time derivative of the radius vector r from the origin of the coordinate system:

$$a = \frac{d^2 r}{dt^2} \qquad (2)$$

Thus, coordinate differences between stations P_1 and P_2 can be determined by doubly integrating a

$$r_2 - r_1 = \int_{t_1}^{t_2}\int_{t_1}^{t_2} a\,dt\,ds \qquad (3)$$

where dt and ds both refer to time. This is the basic equation underlying inertial navigation. To use it for positioning purposes, an accelerometer triad has to be mounted on a vehicle in such a way that its orientation in space is known at each instant of time. While the vehicle moves from P_1 to P_2, accelerations are measured and integrated along the three orthogonal axes. Because Newton's laws of motion are only valid in inertial coordinate systems, r_2 and r_1 strictly represent three-dimensional Cartesian vectors from the origin of the inertial system. However, it can be shown that, for measurements on the surface of the earth, the origin can be moved to the center of gravity of the earth. The errors produced by this approximation are at least one order of magnitude smaller than the measuring errors.

The orientation of the accelerometer triad can be achieved by determining all orientation changes of the measuring unit against inertial space (strapdown systems) or by controlling the orientation changes in such a way that they can be described by a well defined mathematical function (gimballed systems). This second possibility is used in all inertial surveying systems presently available. It requires the mechanical implementation of a so-called stable element - a stabilized platform that isolates the measuring unit from the vehicle motions. In the space-stabilized system, the platform frame is fixed with respect to inertial space; in the local-level system, it rotates against inertial space in such a way that the z-axis of the measuring system remains orthogonal to an initially defined ellipsoid. This second system can be related in a very simple way to the ellipsoidal coordinate systems used in geodesy. The transformation from the ellipsoidal coordinates to the conformal mapping systems and the separation of the height component are well known.

The stabilization of the platform is done by a system of gyroscopes, i.e., by highly refined spinning tops. A gyroscope maintains its orientation in space unless acted upon by a torque. Thus, a system of three orthogonal axes defined by gyroscopes at the start of the mission can be maintained independent of vehicle motions. This frame, which can be transported, is ideally suited to define the orientation of the three accelerometer axes.

An alignment precedes each inertial survey to relate the orientation of the measuring frame to that of the coordinate system, in which the control points are given. During this procedure, the measuring unit is leveled and oriented by gyro-compassing. The leveling aligns the z-axis of the system along the local plumb line. If the deflections of the vertical are known at the initial point, the transformation to an ellipsoidal system of coordinates can be done in a simple manner. Gyrocompassing is based on precession, i.e., on the specific movement the spin axis of the gyro undergoes when a torque is applied perpendicular to it. The same principle is used in gyrotheodolites to establish astronomic north.

Two problems arise when applying Formulas 1 and 3 to systems moving on the surface of the earth or its vicinity. First, the measuring unit moves in the gravity field of the earth and here Formula 1 has to be replaced by

$$f = a - g \qquad (4)$$

where, for convenience, $m = 1$ has been assumed. Formula 4 shows that to integrate vehicle acceleration the effect of gravitational attraction has to be subtracted first from the measurements. This is not difficult for the normal component, which comprises about 99 percent of total gravitation. However, it is more complicated for the gravity disturbance vector, or local gravity anomaly. The second difficulty to be overcome is that the measuring unit moves on a body which itself rotates with respect to inertial space: thus, the transformations between the acceleration vector in an inertial frame and one in a uniformly rotating frame have to be introduced into Equation 3. The final expression is of the form

$$r_2 - r_1 = \int_{t_1}^{t_2}\int_{t_1}^{t_2} (f(r,\dot{r},\ddot{r},\Omega) \;+ g) \;\; dt \; ds \qquad (5)$$

where $(r,\dot{r},\ddot{r},\Omega)$ indicates that the acceleration now depends on r, its first and second time derivatives $\dot{r}$ and $\ddot{r}$, and on a skew-symmetric matrix Ω containing angular velocities. The vector r now refers to a topocentric system. Again $m = 1$ has been assumed.

Thus, inertial positioning can be summarized as the determination of vehicle accelerations from measurements and their integration along three orthogonal axes whose orientation with respect to an inertial frame is known at any instant of time.

So far, it has been assumed that the measuring unit produces errorless data. The abstract mathematical model is necessary to approximate, as closely as possible, the physical process under ideal conditions. In reality, measuring errors must always be taken into account and optimal procedures must be found to eliminate them. In the inertial surveying systems, all major errors are time dependent and show a very typical behavior over certain periods of time.

The "on-board" computer rides 'shortgun' and provides initial real time results. One feature of this sophisticated "machine" is the so called Kalman Filter: the theory of this smoothing analysis is not covered here. Essentially, it performs a statistical filtering of the raw data to obtain the best estimates, by a process of weighting all new data on the basis of the characteristics of all previous measurements and a prior knowledge of the nature and usual magnitude of the errors. In other words, the computer determines rough coordinates from the filtered observations as each new station is surveyed. When closure of the inertial traverse is made on a known station, the computer will distribute the accumulated error to the rough coordinates by an approximate adjustment procedure. These approximate coordinates are for field use only, as the final adjusted coordinates will be determined by a more rigorous reduction and adjustment procedure performed on a larger main frame computer, using cross-over points, for example, as constraints .

The data, stored on magnetic tape, include the rough and approximate coordinates, estimates of changes in the deflection of the vertical, estimates of changes in the gravity anomaly, and all other monitored and computed data required for more detailed studies and adjustments. Final processing and adjustment of the observed data result in a total solution to the geodetic problem; changes in the directions of the vertical, changes in the gravity anomaly, latitude, and elevation may all be determined.

At this early stage in this new, exciting, technology, the average land-surveyor is ill-equipped mathematically to do other than regard the on board computation facility as a "black-box" very much the province of the computer expert.

At the risk of being over simplistic, the following elementary summary can be made of the major characteristics of inertial surveying systems:-

Essentially the system consists of three basic components;

- gyroscopically stabilized inertial platform, whose three orthogonal axes are oriented in space to local east, local north and local vertical;
- sensitive accelerometers which measure accurately any displacement of the platform along the three axes; and
- associated computing equipment to interpret the accelerometer data and convert it into meaningful geodetic data, in terms of latitude, longitude and elevation. The system is opened and closed at known control points and is best used in an area mode so that constraining cross-over points can be used in the subsequent lease squares adjustment. In such a system the curvature of the earth and anomalous gravity result in rapid accumulation of error: accelerometer errors grow with $(\text{time})^2$ and a gyro drift error grows with $(\text{time})^3$. If the transporting helicopter or vehicle is frequently stopped e.g. at 4 - 5 minute (e.g. 2 km) intervals, the computer can reset the velocity to zero (zero velocity update, ZUPT) and remove the accumulated velocity errors.

Thus I.S.S. suffers from obvious disadvantages:-

(a) the requirement for a crossing network of traverse lines;

(b) the need for ZUPT at frequent intervals;

(c) the need for close landing (helicopter) positions, or ready vehicular access, for all control points.

It is worth emphasising, however, that it is the only method free from atmospheric effects, and, of course, line-of-sight is not required. Similarities between I.S.S. and geodetic satellite surveys include the necessity for information about the earth's gravity field in which the sensor moves. Procedures to reduce systematic errors are principally ZUPT, and use of short arc or translocation techniques respectively, whilst on-board gradiometric methods are now being used for the determination of "g" in equation 5. The method appears to have a bright future, and helicopter-borne operation could be very significant for countries like P.N.G. where vehicular modes of transport are very limited.

APPENDIX 15

UNIVERSITY OF QUEENSLAND - BACHELOR OF SURVEYING PROGRAM AND SYLLABI OF MATHEMATICS COMPONENTS

FIRST SCHEDULE - UNDERGRADUATES

Subject – PH

Semester 1 (1st)

GE232	5 CC	Thematic Cartography
ME100	7 CC	Engineering Mathematics 1A
ME101	8 CC	Engineering Mathematics 1B
PH100	10 CC	Mechanics, General & Thermal Physics I
PH180	4 CC	Experimental Physics: General I
PH124	7 EE	Astronomy I
SY103	5 CC	Surveying Principles
SY115	10 CC	Surveying Mathematics I

Credit point totals PC-49 PE-7 HC-49 HE-7

Semester 2 (2nd)

ES101	1 CC	First Aid
ME102	8 CC	Engineering Mathematics 1C
ME104	5 CC	Engineering Mathematics 1D(a)
ME105	3 EE	Engineering Mathematics 1D(b)
PH101	10 CC	Electricity, Magnetism & Optics I
PH123	7 EE	Environmental Physics: Atmosphere & Oceans
PH171	5 EE	Experimental Physics: Electrical 1C
SY123	8 CC	Surveying Instrumentation & Procedure
SY131	4 CC	Survey Plans

Credit point totals PC-36 PE-15 HC-36 HE-15

Semester 3 (1st)

CS100	8 CC	Introduction to Computer Sc.
E2209	3 CC	Surveying Hydraulics
ME200	8 CC	Engineering Maths IIA
ME203	6 CC	Engineering Maths IID Statistics
PH210	5 EE	Physical Optics II
PH273	6 EE	Experimental Optics
SY216	5 CC	Surveying Maths II
SY241	5 CC	Route Surveys
SY242	5 CC	Mine Surveys
SY245	4 CC	Atmospheric Surveys

Credit point totals PC-44 PE-11 HC-44 HE-11

Semester 4 (2nd)

GM100	9 CC	Geology IE
GM111	3 CC	Geology IE - Practical
SY212	6 CC	Projective Geometry & Matrix Algebra
SY214	5 CC	Survey Computing
SY215	6 CC	Introduction to Errors & Adjustment
SY244	5 CC	Electromagnetic Distance Measurement
SY261	8 CC	Positional Astronomy
SY281	8 CC	Survey Project A

Credit point totals PC-50 PE-0 HC-50 HE-0

Semester 5 (1st)

AG211	6 CC	Soil & Land Utilisation A
SY341	7 CC	Cadastral Surveys
SY344	5 EE	Survey Instrument Design
SY350	6 EE	Geodetic Mathematics
SY351	7 CC	Geodesy
SY352	9 CC	Geodetic Survey
SY373	6 EE	Photogrammetric Project
SY374	15 CC	Photogrammetry
SY383	3 CC	Seminars

Credit point totals PC-47 PE-17 HC-47 HE-17

Semester 6 (2nd)

AG212	6 CC	Soil and Land Utilisation B
E2414	4 CC	Engineering Construction
RT293	8 CC	Regional & Town Planning
SY331	9 CC	Cartography
SY343	6 CC	Hydrographic Surveys
SY345	4 EE	Workshop Surveys
SY373	6 EE	Photogrammetry C
SY381	6 CC*	Survey Project B
SY382	10 CC	Law, Professional Practice & Valuation

Credit point totals PC-49 PE-10 HC-49 HE-10

*Adequate performance in practical work and/or assignment work in SY341 will satisfy the prerequisits requirements.

LEGEND

CC:	Compulsory Pass/Honours
EE:	Elective
PC:	Pass degree, compulsory subjects
PE:	Pass degree, elective subjects
HC:	Honours degree, compulsory subjects
HE:	Honours degree, elective subjects

SECOND SCHEDULE – HONOURS

Subject – H

Semester 1 (1st)

GR101	10 E	*Inc* Senior German or GR105 or 106 or 107 or XR100 or Intro. German Lang. or Prep. German
GR106	8 E	*Inc* GR101 or 102 or Senior German or Introd. German Lang.
SY411	8 C	Observations H1
SY451	10 E	Observations H2
SY471	(7) E	Photogrammetry IV
SY481	(6) E	Remote Sensing H
SY483	(2) E	Seminar
SY491	(8) C	Honours Thesis
SY796	(5) E	Numerical Photogrammetry

Credit point totals:PC-0 PE-0 HC-16 HE-48

Semester 2 (2nd)

GR101	10 E	*Inc.* Senior German or GR105 or 106 or 107 or XR100 or Intro. German Lang. or Prep. German
GR102	10 E	*Pre* GR101 *Inc* Senior German or GR105 or 106 or 107 or XR100 or Introd. German Lang. or Prep. German
GR107	8 E	*Pre* GR106 *Inc* GR101 or 102 or 105 or Senior German or Introd. German Lang.
SY412	5 E	Observations H2
SY452	6 E	Physical Geodesy II
SY453	6 E	Satellite Geodesy
SY454	6 E	Geodesy IV
SY471	(8) E	Photogrammetry IV
SY481	(6) E	Remote Sensing H
SY483	(1) E	Seminar
SY491	(9) C	Honours Thesis
SY796	(5) E	Numerical Photogrammetry

Credit point totals PC-0 PE-0 HC-9 HE-71

ME100 Engineering Mathematics IA
#7 (2L1T) 1st
Limits & continuity, derivative, techniques of differentiation, maxima & minima, partial derivatives, indefinite integrals. Taylor's theorem.

ME101 Engineering Mathematics IB
#8 (2L1T) 1;2
Vectors & kinematics, including cross products, scalar products & triple products, vector functions, radial & transverse components. Line integrals & potential energy. Particle dynamics including applications to projectiles, circular motion, simple harmonic motion & collisions.

ME102 Engineering Mathematics IC
#8 (2L1T) 2nd Comp: ME100 or MP101
Definite integral, improper integrals, numerical integration, approximations, sequences & series. Taylor series, co-ordinate geometry of conic sections.

ME103 Engineering Mathematics ID
#8 (2L1T) 2nd
(A) Algebra: complex numbers, determinants, matrices, inversion of matrices & simultaneous equations. (B) Applied differential equations: solution of first- & second-order differential equations with application to physical & chemical systems.

ME104 Engineering Mathematics ID(a)
#5 (2C) 2nd
Section A of ME103.

ME105 Engineering Mathematics ID(b)
#3 (1C) 2nd Comp: ME104
Section B of ME103.

ME200 Engineering Mathematics IIA
#8 (2L1T) 1st Pre: ME100 + 103
Functions of several variables; multiple integrals. Taylor series, maxima & minima. Lagrange multipliers. Matrix theory; eigen values & eigenvectors, reduction to diagonal form, applications to simultaneous linear differential equations. Numerical methods

ME203 Engineering Mathematics IID (Statistics)
#6 (2C) 1;2 Pre: ME100 + 102
Discrete & continuous probability distributions Treatment of experimental data, the t, F & chi squared tests. Regression analysis. Analysis of variance. Quality control.

APPENDIX 16

COURSE STRUCTURE FOR THE PNGUT DIPLOMA/BACHELOR OF TECHNOLOGY IN SURVEYING

DIPLOMA IN SURVEYING

FIRST PROFESSIONAL YEAR

Grade 12 Entry common for all Surveying, Survey Drafting and Cartography.

FIRST SEMESTER

Code		Subject	Average Weekly Hours
MA	151	Mathematics IIS	6
LA	136	English for Surveying and Cartography I	3
SV	101	Linear Measurement	4
SV	122	Survey Calculations I	4
SV	131	Survey Drafting I	4
SV	103	Levelling	5
			26

SECOND SEMESTER

Code		Subject	Average Weekly Hours
MA	152	Mathematics IIS	5
LA	136	English for Surveying and Cartography I	3
SV	104	Theodolite Surveying I	5
SV	102	Tacheometric Surveying	3
SV	132	Survey Drafting II	5
SV	106	Map Reading	2
SV	108	First Aid	1
SV	123	Introduction to Calculator Programming.	2
			26

SV 151 SURVEY CAMP I — All students to attend a 2-week camp out of semester time.

SECOND PROFESSIONAL YEAR

FIRST SEMESTER

Code		Subject	Average Weekly Hours
SV	203	Engineering Survey Theory	3
SV	206	Engineering Survey Practical	4
SV	221	Sun Azimuth Theory	2
SV	223	Sun Azimuth Practical	2
SV	202	Theodolite Surveying II	5
SV	222	Engineering Survey Computations	5
SV	231	Engineering Survey Drafting	3
SV	293	Mechanical Equipment	1
			25

BEST COPY AVAILABLE

SECOND SEMESTER

SV	224	Land and Implementation	3
SV	225	Cadastral Practice I	3
SV	226	Cadastral Field Survey I	3
SV	284	Cadastral Drafting I	7
SV	207	Customary Land Tenure	2
SV	228	Measurement Assessment	3
SV	229	Photo Interpretation	4
			25

SV 251 SURVEY CAMP II — All students to attend a 2-week camp out of semester time.

PRACTICAL TRAINING YEAR

SV 300 INDUSTRIAL TRAINING

BACHELOR OF TECHNOLOGY (SURVEYING)
THIRD PROFESSIONAL YEAR
FIRST SEMESTER

MA	451	Mathematics IIIS	4
AR	360	Urban Planning	3
LA	436	English for Surveying and Cartography II	2
SV	406	Geodetic Computations I	4
SV	401	Control Surveys I	4
SV	404	Cadastral Project	4
SV	431	Photogrammetry I	4
			25

SECOND SEMESTER

AR	360	Urban Planning	3
LA	436	English for Surveying and Cartography II	2
SV	412	Professional Practice	1
SV	405	Introduction to Satellite Geodesy and Remote Sensing	2
SV	408	Geodetic Computations II	4
SV	402	Control Surveys II	4
SV	421	Adjustment Computations	3
SV	446	Valuation for Surveyors	2
SV	432	Photogrammetry II	2
			23

SV 451 SURVEY CAMP II — All students to attend a 2-week camp out of semester time.

BASIC MATHEMATICS COMPONENT OF THE PNGUT BACHELOR OF TECHNOLOGY IN SURVEYING

MA 151 (MATHEMATICS IS)

Hours per week:	6 (first semester)
Corequisites/prerequisites:	MA 050 or equivalent
Syllabus:	Revision of algebra and functions. Differentiation. Approximation of areas by trapezoidal and Simpson's rule. Introduction to partial differentiation and matrices.
Textbooks:	Mathematics Department Learning Units
Reference:	Stroud, K.A., *Engineering Mathematics*, Second Edition (London, Macmillan, 1982)
Assessment:	Continuous assessment — 60% Written examination — 40%.

MA 152 (MATHEMATICS IS)

Hours per week:	5 (2nd semester)
Corequisites/prerequisites:	MA 050 Or equivalent
Syllabus:	Geometry of lines, angles, polygons, circles and their intersection. Introductory geometry of three dimensions and geometric transformations.
Textbook:	Dube, L. and Lyons, K., *Geometry for Surveyors*, (Lae, MEC, 1981)
Assessment:	Continuous assessment — 60% Written examination — 40%.

MA 351 (MATHEMATICS 3S)

Hours per week:	4 (first semester)
Prerequisite:	MA 152
Syllabus:	Matrix algebra, addition, multiplication, transpose, inverse; solution of simultaneous equation, Taylor series. Differentials applied to error problems in surveying.
Textbook:	Stroud, K.A., *Engineering mathematics*, (London, Macmillan, 1970)
Examination:	Continuous assessment — 50% Written examination — 50% (1 x 3 hours).

BEST COPY AVAILABLE

AN EXPERIMENTAL APPROACH TO TEACHING MATHEMATICS

Donald Joyce
University of Papua New Guinea

Abstract

Many students feel that mathematics has all been "done" and their task is to memorise it. They are surprised to hear that mathematics is still being created and unable to imagine themselves "making" mathematics. This paper describes an approach, combining experiment and theory, which has been used with some success in teaching introductory courses in numerical analysis and can be applied in many other areas of mathematics. It is claimed that this approach gives the student much better motivation and insight and allows the mathematics to be developed naturally, as it is needed, rather than having to be "pulled out of a hat."

Introduction

Modern introductory numerical analysis texts tend to fall into one of two classes: those which describe numerical methods and illustrate their application by numerical examples and computer programs, and those which analyse numerical methods and present a series of theorems and proofs. Neither approach is very satisfactory: the first failing to provide a real understanding of how methods work (or fail to work), the second failing to provide motivation or insight.

This paper suggests ways in which these shortcomings may be overcome by combining experiment and theory. Apart from its advantages for teaching, this third approach is more realistic since much of the theory of numerical analysis has been developed with the aid of numerical experimentation. It will be illustrated by showing how experiment and theory may be used interactively when investigating the convergence of iterative methods for solving nonlinear equations.

Fixed Point Iterations

One class of methods for solving the nonlinear equation

$$f(x) = 0 \tag{1}$$

involves iterations of the form

$$x_{n+1} = g(x_n)\ , \tag{2}$$

where the desired root α is a fixed point of the iteration function g

i.e.

$$\alpha = g(\alpha) \tag{3}$$

A suitable g may be found by trial and error.

For example, the cubic

$$f(x) = x^3 - x - 1 \tag{4}$$

may be shown to have one real zero, $\alpha \approx 1.32472$. The equation

$$x^3 - x - 1 = 0 \tag{5}$$

may be rearranged in various ways to give equations of the form

$$x = g(x)\ . \tag{6}$$

Four such rearrangements are

$$x = x^3 - 1 \tag{7}$$

$$x = (x + 1)^{\frac{1}{3}} \text{ (from } x^3 = x + 1)\ , \tag{8}$$

$$x = (1 + \frac{1}{x})^{\frac{1}{2}} \text{ (from } x^2 = \frac{x + 1}{x})\ , \tag{9}$$

$$x = \frac{1}{x} + \frac{1}{x^2} \text{ (from } x = \frac{x + 1}{x^2})\ . \tag{10}$$

Table I shows the results obtained when each of these rearrangments is used as the basis of an iteration of the form (2) with $x_0 = 1$.

$g(x)$	x_0	x_1	x_2	x_3	x_4	x_5
$x^3 - 1$	1	0	-1	-2	-9	-730
$(x+1)^{\frac{1}{3}}$	1	1.25992	1.31229	1.32235	1.32427	1.32463
$(1+\frac{1}{x})^{\frac{1}{2}}$	1	1.41421	1.30656	1.32867	1.32387	1.32490
$\frac{1}{x} + \frac{1}{x^2}$	1	2	0.75	3.11111	0.42474	7.89734

Table I - Iterates

The four types of behaviour observed may be classified as monotonic divergence (7), monotonic convergence (8), oscillatory convergence (9), and oscillatory divergence (10), and are made more obvious in an error table (Table II), in which

$$e_n = \alpha - x_n \tag{11}$$

$g(x)$	e_0	e_1	e_2	e_3	e_4	e_5
$x^3 - 1$	0.325	1.32	2.32	3.32	10.3	731
$(x+1)^{\frac{1}{3}}$	0.325	0.0648	0.0124	0.00237	0.00045	0.00009
$(1+\frac{1}{x})^{\frac{1}{2}}$	0.325	-0.0895	0.0182	-0.00395	0.00085	-0.00018
$\frac{1}{x} + \frac{1}{x^2}$	0.325	-0.675	0.575	-1.79	0.900	-6.57

Table II - Errors

We would like to uncover the mechanism which determines the type of error behaviour displayed. In the hope that some sort of pattern may emerge, we compute the ratios of successive errors (Table III).

$g(x)$	e_1/e_0	e_2/e_1	e_3/e_2	e_4/e_3	e_5/e_4
$x^3 - 1$	4.06	1.76	1.43	3.10	7.10
$(x+1)^{\frac{1}{3}}$	0.199	0.191	0.191	0.190	0.200
$(x+\frac{1}{x})^{\frac{1}{2}}$	-0.275	-0.203	-0.217	-0.215	-0.212
$\frac{1}{x}+\frac{1}{x^2}$	-2.08	-0.852	-3.11	-0.503	-7.30

Table III - Error Ratios

No obvious pattern can be detected in the divergent cases, but the behaviour in the convergent cases suggests the hypothesis:

$$\frac{e_{n+1}}{e_n} \to k \quad \text{as} \quad n \to \infty \tag{12}$$

Now we are motivated to use some mathematical analysis. From (2), (3) and (11) we obtain

$$\frac{e_{n+1}}{e_n} = \frac{g(\alpha) - g(x_n)}{\alpha - x_n} \tag{13}$$

and we call on the mean value theorem for derivatives in order to write

$$\frac{e_{n+1}}{e_n} = g'(\beta_n) \ , \tag{14}$$

where β_n is between α and x_n .

If the method is convergent, so that $x_n \to \alpha$ as $n \to \infty$, then

$$\frac{e_{n+1}}{e_n} \to g'(\alpha) \text{ as } n \to \infty, \tag{15}$$

which confirms our hypothesis (12). Thus, if $g'(\alpha) \neq 0$, the convergence will be asymptotically linear and

$$e_{n+1} \approx ke_n, \tag{16}$$

where $k = g'(\alpha)$

is the asymptotic convergence factor. The four types of convergence behaviour correspond to $g'(\alpha) > 1$, $0 < g'(\alpha) < 1$, $0 > g'(\alpha) > -1$ and $g'(\alpha) < -1$, respectively.
From (16) we deduce $e_1 \approx ke_3$, $e_2 \approx ke_1 \approx k^2e_0$, $e_3 \approx ke_2 \approx k^3e_0$, which lead us to prove (by induction)

$$e_n \approx k^n e_0 \tag{17}$$

We can deduce that the iteration (2) will converge to α if $|g'(\alpha)| < 1$ and x_0 is sufficiently close to α. We can go further and use "epsilon-and-delta analysis", to prove a fixed point theorem:
If there exists an interval $I = [a, b]$ such that, for all x in I, $g(x)$ is in I and $|g'(x)| < 1$, then $g(x)$ has exactly one fixed point α in I and $x_{n+1} = g(x_n)$ converges to α for all x_0 in I.

Returning to our example, if we define

$$g_1(x) = x^3 - 1$$
$$g_2(x) = (x+1)^{\frac{1}{3}}$$
$$g_3(x) = (1 + \frac{1}{x})^{\frac{1}{2}}$$
$$g_4(x) = \frac{1}{x} + \frac{1}{x^2}$$

and use $\alpha^3 = \alpha + 1$

and $\alpha^2 = 1 + \frac{1}{\alpha}$

small. The optimum choice is $c = -1/f'(\alpha)$, for which $g'(\alpha) = 0$. However we will not normally know $f'(\alpha)$, so we are led to investigate the iteration

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad (19)$$

which has $$g(x) = x - \frac{f(x)}{f'(x)} \qquad (20)$$

and is called the Newton-Raphson method. Applied to the cubic (4) with $x_0 = 1$, the method produces the results shown in Table IV:

n	0	1	2	3	4
x_n	1	1.5	1.34783	1.32520	1.32472
e_n	0.32472	-0.17528	-0.02311	-0.00048	0
e_{n+1}/e_n	-0.541	0.132	0.0208	0	

Table IV - Newton-Raphson

In this case we have the obvious hypothesis:

$$\frac{e_{n+1}}{e_n} \to 0 \text{ as } n \to \infty \qquad (21)$$

which fits in with our general result (15), since

$$g'(\alpha) = 1 - \frac{f'(\alpha)}{f'(\alpha)} + \frac{f(\alpha)f''(\alpha)}{f'(\alpha)f'(\alpha)} = 0$$

(unless $f'(\alpha) = 0$, in which case we can use de l'Hôpital's rule to obtain the revised iteration function

$$g(x) = x - \frac{2f(x)}{f'(x)}$$

for which $g'(\alpha) = 0$, provided $f''(\alpha) \neq 0$).

We can obtain more information about the convergence of the Newton-Raphson method by using Taylor's Theorem in the form:

$$g(x_n) = g(\alpha) + (x_n - \alpha)\, g'(\alpha) + \frac{1}{2}(x_n - \alpha)^2\, g''(\gamma_n)\,, \qquad (22)$$

with γ_n between α and x_n. From (11) and (13) we find that

$$\frac{e_{n+1}}{e_n} = g'(\alpha) - \frac{1}{2} e_n\, g''(\gamma_n)$$

so that $$\frac{e_{n+1}}{e_n{}^2} = -\frac{1}{2}\, g''(\gamma_n)$$

if $$g'(\alpha) = 0\,.$$

If the method is convergent, so that $x_n \rightarrow \alpha$ as $n \rightarrow \infty$

then $$\frac{e_{n+1}}{e_n{}^2} \rightarrow -\frac{1}{2}\, g''(\alpha) \quad \text{as } n \rightarrow \infty \qquad (23)$$

Thus, if $g'(\alpha) = 0$, the convergence will be asymptotically quadratic and $$e_{n+1} \approx K e_n^2 \qquad (24)$$

where $$K = -\frac{1}{2} g''(\alpha)$$

From (24) we deduce that $e_1 \approx K e^2{}_0$, $e_2 \approx K e_1^2 \approx K^3 e_0^4$, $e_3 \approx K e_2^2 \approx K^7 e_0^8$,

which leads us to prove (by induction again)

$$K e_n \approx (K e_0)^{2^n} \qquad (25)$$

We can now tabulate the ratios e_{n+1}/e_n^2 (Table V) in the expectation that they should tend to a constant, until rounding errors become significant.

n	0	1	2	3
e_{n+1}/e_n^2	-1.66	-0.752	-0.903	0

Table V - Newton-Raphson Ratios

The "theoretical value" can be calculated from

$$g'(x) = f(x)h(x) ,$$

where $$h(x) = \frac{f''(x)}{f'(x)f'(x)} .$$

We obtain $$g''(\alpha) = f'(\alpha)h(\alpha) + f(\alpha)h'(\alpha) = \frac{f''(\alpha)}{f'(\alpha)}$$

provided $$f'(\alpha) \neq 0 .$$

Thus $$K = - \frac{f''(\alpha)}{2f'(\alpha)}$$

which is approximately -0.932 for this example. Our experimental results are not inconsistent with the theory, but the rapid convergence does not allow the behaviour to "settle down" before the desired accuracy is achieved.

Secant Rule

If $f'(x)$ is difficult to compute, we can use the approximation

$$f'(x_n) \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

to obtain the Secant Rule:

$$x_{n+1} = x_n - f(x_n) \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} . \qquad (26)$$

Applied to the cubic (4), with $x_{-1} = 1$ and $x_0 = 2$, the method produces the results shown in Table VI:

n	-1	0	1	2	3	4	5
x_n	1	2	1.16667	1.25311	1.33721	1.32385	1.32471
e_n	0.32472	-0.67528	0.15805	0.07161	-0.01249	0.00087	0.00001
e_{n+1}/e_n	-2.08	-0.234	0.453	-0.174	-0.0697	0.0115	
e_{n+1}/e_n^2	-6.40	0.347	2.87	-2.43	5.58	13.2	

Table VI - Secant Rule

We are led to the same hypothesis (21) as for Newton-Raphson (this behaviour is known as superlinear convergence), but now we cannot use (15) since x_{n+1} depends on x_{n-1} as well as x_n. This double dependence, combined with the behaviour of e_{n+1}/e_n and e_{n+1}/e_n^2, leads us to tabulate the ratios $e_{n+1}/e_n e_{n-1}$ (Table VII):

n	0	1	2	3	4
$e_{n+1}/e_n e_{n-1}$	-0.721	-0.671	-1.10	-0.973	-0.920

Table VII - Secant Rule Ratios

Since the last two ratios look rather like -0.932, we make a "stab in the dark":

$$\frac{e_{n+1}}{e_n e_{n-1}} \to K \quad \text{as} \quad n \to \infty \tag{27}$$

This time we will need some results about divided differences to confirm our hypothesis.

Since
$$x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})} \tag{28}$$

we can deduce that
$$e_{n+1} = \frac{\alpha f(x_n) - \alpha f(x_{n-1}) - x_{n-1} f(x_n) + x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$= \frac{e_{n-1} f(x_n) - e_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$$

so
$$\frac{e_{n+1}}{e_n e_{n-1}} = -\frac{\dfrac{f(x_n)-f(\alpha)}{x_n-\alpha} - \dfrac{f(x_{n-1})-f(\alpha)}{x_{n-1}-\alpha}}{x_n - x_{n-1}} \, \frac{x_n - x_{n-1}}{f(x_n)-f(x_{n-1})}$$

$$= -\frac{f''(\eta_n)}{2} \, \frac{1}{f'(\zeta_n)} ,$$

where η_n is in the interval spanned by α, x_n and x_{n-1}, and ζ_n is between x_n and x_{n-1}. Thus if the method is convergent, so that x_n and $x_{n-1} \to \alpha$ as $n \to \infty$,

then
$$\frac{e_{n+1}}{e_n e_{n-1}} \to -\frac{f''(\alpha)}{2f'(\alpha)} \quad \text{as} \quad n \to \infty \tag{29}$$

just as we predicted. Once again the experimental results are not inconsistent with the theory; in fact the fit is much better than for Newton-Raphson.

Inspired by our success in obtaining the error formula (25) for Newton-Raphson, we try to find a corresponding result for the Secant Rule.

From
$$\frac{e_{n+1}}{e_n e_{n-1}} \approx K$$

we obtain
$$K\, e_{n+1} \approx (Ke_n)(Ke_{n-1})$$

and hence
$$\ln\,(Ke_{n+1}) \approx \ln\,(Ke_n) + \ln\,(Ke_{n-1})\,.$$

If we set
$$E_n = \ln\,(Ke_n),\ n = 0,1,2,\ \ldots$$

we have an approximate difference equation:

$$E_{n+1} \approx E_n + E_{n-1}$$

which should have a solution something like

$$E_n \approx p\left(\frac{1+\sqrt{5}}{2}\right)^n + q\left(\frac{1-\sqrt{5}}{2}\right)^n$$

For large n , $$E_n \approx p\left(\frac{1+\sqrt{5}}{2}\right)^n$$

so that $$E_{n+1} \approx \left(\frac{1+\sqrt{5}}{2}\right) E_n$$

and $$Ke_{n+1} \approx (Ke_n)^{\frac{1+\sqrt{5}}{2}}$$

From this we can deduce that

$$Ke_n \approx (Ke_0)^{\left(\frac{1+\sqrt{5}}{2}\right)^n} \qquad (30)$$

So Fibonacci numbers and the golden section have "popped up" once again!

Regula Falsi

The Secant Rule does not always converge, so we may be tempted to use a safer approach based on (28), in which we attempt to enclose the root in a shrinking interval. The method is known as Regula Falsi or the Rule of False Position and is based on the iteration

$$x_{n+1} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$$

with $$b_{n+1} = x_{n+1} \text{ and } a_{n+1} = a_n \text{ if } f(x_{n+1})f(b_n) > 0 ,$$

and $$b_{n+1} = b_n \text{ and } a_{n+1} = x_{n+1} \text{ otherwise.}$$

If $f(a_0)f(b_0) < 0$ initially and $f(x)$ is continuous, we should always have $f(a_n)f(b_n) \leq 0$. Applied to the cubic (4), with $a_0 = 1$ and $b_0 = 2$, the method produces the results shown in Table V[illegible] (where we have set $e_n = \alpha - a_n$, since b_n does not change):

n	0	1	2	3	4
a_n	1	1.16667	1.25311	1.29344	1.31128
b_n	2	2	2	2	2
e_n	0.32472	0.15805	0.07161	0.03128	0.01344
e_{n+1}/e_n	0.487	0.453	0.437	0.430	

Table VIII - Regula Falsi

The method appears to be behaving like a fixed point iteration with $\frac{e_{n+1}}{e_n} \to \kappa$ as $n \to \infty$. We can verify this if we observe that the method is equivalent in this case to

$$x_{n+1} = x_n - f(x_n) \frac{x_n - b}{f(x_n) - f(b)}$$

so that $g(x) = x - f(x)d(x)$,

with $d(x) = \dfrac{x - b}{f(x) - f(b)}$,

and
$$\begin{aligned} g'(\alpha) &= 1 - f'(\alpha)d(\alpha) - f(\alpha)d'(\alpha) \\ &= 1 + f'(\alpha)\frac{\alpha - b}{f(b)} \end{aligned}$$

For this example, $g'(\alpha) \approx 0.424$, so that the actual errors have behaved in accordance with the theory.

It would be nice to show that the error behaviour is related to that of the Secant Rule, so we use Taylor's Theorem again to substitute

$$f(b) = f(\alpha) + (b - \alpha)f'(\zeta)$$

and
$$f(b) = f(\alpha) + (b - \alpha)f'(\alpha) + \frac{1}{2}(b - \alpha)^2 f''(\nu)$$

(where ζ and ν lie between α and b) in the denominator and numerator respectively of

$$\varkappa \quad = \quad \frac{f(b) \; - \; (b - \alpha)f'(\alpha)}{f(b)}$$

to obtain

$$\varkappa \quad = \quad \frac{(b - \alpha)f''(\nu)}{2f'(\zeta)} \; .$$

Thus

$$\frac{e_{n+1}}{e_n e_{-1}} \approx - \frac{f''(\nu)}{2f'(\zeta)} \; ,$$

where

$$e_{-1} \; = \; \alpha \; - b \; .$$

It is not hard to show that the end point with positive (negative) function value remains fixed if $f''(x)$ is positive (negative) throughout the interval. In such cases, Regula Falsi is a fixed point iteration with

$$g'(\alpha) \; = \; 1 + f'(\alpha)\frac{e_{-1}}{f(x_{-1})} \; ,$$

where x_{-1} is the fixed end point and $e_{-1} = \alpha - x_{-1}$.

Conclusion

We have seen that, when investigating the convergence of iterative methods, we can use numerical experiments to suggest theoretical results, which may then be confirmed using mathematical analysis, and which may suggest further experiments, and so on. It is claimed that this approach gives the student much better motivation and insight, and allows the mathematics to be brought into play naturally, as it is required, rather than having to be "pulled out of a hat". The area of nonlinear equations lends itself particularly readily to the use of this approach, but other areas of numerical analysis may profitably be treated in the same way, and the approach may also be found useful in teaching mathematical analysis, abstract algebra, applied mathematics, indeed any area of mathematics.

Fourth National Conference on

Mathematics Education, May, 1984

Papua New Guinea University of Technology

Mathematics Education Centre

MINIMAL REMEDIATION: PRE-TEST AS A KICK IN THE PANTS

John Gough
Lecturer in Mathematics Education
University of Papua New Guinea
(Using tests devised by Angela Balkwill,
Coordinator of Preliminary Year
Mathematics, UPNG.

Introduction: Sullivan's Pioneering Study

At the Third National Conference on Mathematics Education, held at the Mathematics Education Centre of the PNG University of Technology in May 1983, Peter Sullivan who was then working as a part-time research officer in the MEC announced some exciting experimental results (since published as **MEC Report Number 28: Remediation of Basic Arithmetic Weaknesses in Post-Secondary Students** Mathematics Education Centre; Lae, 1983). In an elegant small-scale experiment carried out early in 1983, Sullivan had taken a group of 44 ex-Grade 10 students who were beginning a Pre-Employment Technical course.

Part of this course was a Basic Skills Course in arithmetic and basic mathematics designed to revise mathematics already taught (and possibly learned) in grades 1 to 10, and to support the mathematical skills needed for other vocational courses that were part of the students' total training.

Sullivan administered a one-hour pre-test to the whole group, and then randomly allocated each student to one of four groups: one group completed the normal Basic Skills Course taught by a mathematics lecturer, a second group was given an equivalent arithmetic course taught by a trade lecturer, a third group's only treatment was to have the errors on their pre-test papers carefully corrected and explained to each student on a one-to-one basis, and a fourth group did nothing further except normal trade courses, and so acted as a control for the other groups. All students knew there would be a similarly constructed post-test at the end of the time allowed for completion of the Basic Skills Course. A delayed second post-test was given 8 weeks after the first post-test to test retention.

Statistical tests showed that there were no significant differences between the four groups for either of the two post-tests. There was a small uniform improvement for all groups between the pre-test and the first post-test, and this small improvement was maintained overall 8 weeks later in the second post-test.

The carefully developed, skilfully taught Basic Skills Course was no better in improving students mathematics skills than no direct mathematics instruction at all! This was startling. The best available remediation was only as good as no remediation.

Sullivan noted at the end of his Report that basic arithmetic programs are used at the start of many post-grade 10 training institution.

> "It would seem appropriate ... to mount a large scale study of this type to further investigate the effectiveness of these programs. If the results confirm the findings of this study, it will mean that a radically different approach to arithmetic teaching will be necessary at the post-secondary level". (Sullivan, 1983, p14)

This paper presents findings of a large-scale study carried out at UPNG at the start of 1984's Preliminary Year Mathematics Course.

UPNG's Preliminary Year Basic Mathematics Study

In the Orientation Fortnight at the start of the 1984 UPNG Preliminary Year (PY) Mathematics Course, Angela Balkwill, the Coordinator of PY maths, and the staff of lecturers and tutors who teach PY maths, had the task of identifying and helping students whose basic mathematics skills were weak. However there was little time available for large amounts of revision coaching, it was not immediately clear how many students really needed a lot of help, and any revision work would have been very boring for the teaching staff and for students who did not need all or some of the revision. Sullivan's study seemed to offer an efficient alternative. Interested to pursue the implications of Sullivan's study, Ms Balkwill designed a one-hour pre-test that would cover the basic entry skills that were assumed to be necessary for PY maths, and that should already have been learned in grades 1 to 10.

This PY test had similarilities to Sullivan's test. But because both tests were screening for entry to very different programs there were differences. The PY test included topics such as Directed Numbers, Order of Operations, and Squares and Square Roots which were not included in Sullivan's test. On the other hand, Sullivan's test included such everyday skills as Clock Reading, Area and Volume, Converting Units of Measurements, and Estimating Sizes, Weights and Volumes, which might have been appropriate but were not included in the PY test.

It should be noted that Sullivan's small group of students was beginning a relatively short-term vocational training program. In contrast the PY group was beginning a one-year subtertiary program which is intended to reach a similar Standard to Grades 11 and 12 at National High School. Successful PY students would gain entry to undergraduate degree programs at UPNG. Also most of Sullivan's students were school leavers who had completed Grade 10 in the previous year. In contrast to this, UPNG actively discourages direct entry to PY from Grade 10 and seeks to enrol mature-age students who completed Grade 10 several years previously, yet failed then to win a place at National High School, or went directly into jobs, or chose to take alternative post-Grade 10 courses, and have subsequently spent several years in the workforce or in village life. Entry to PY usually requires students to have achieved at least all Upper Passes at Grade 10.

Clearly the two populations are different in many ways, as are the test instruments used. Also there are important differences in experimental design, as we shall see. However both situations share the common factor of being tested on ex-Grade 10 basic mathemtics.

The "experimental design" (speaking loosely) of the PY situation was this: students did the pre-test at the start of the Orientation Fortnight. Their papers were marked by individual tutors and handed back the next day. At this time students were told there would be a similar post-test at the end of the Fortnight. This post-test would count towards their final assessment. They were also given a copy of a booklet called Introduction To Preliminary Year Mathematics published by UPNG Mathematic Department) which discussed the maths concepts and skills that were included in the pre-test. Students were advised to use their corrected pre-test paper and the booklet to prepare for the post-test. They were also invited to seek assistance from the PY teaching team, though few students did seek any such assistance. (Unfortunately exactly which students asked for help, and how much help they may have received is not known. However it is certain that the number of students helped and the amount of help given are both ignorable for the purposes of this study.)

Results were scored for Pre-Test and Post-Test Totals, and also for Pre-Test and Post-Test Totals on Sub-Tests on Fractions, Decimals, Directed Numbers, Order of Operations, and Squares and Square Roots.

A total of 96 students sat for both the Pre-Test and the Post-Test. A handful of students sat for only one of the tests and their results had to be ignored. There were 11 females and 85 males in this group.

Unfortunately, because students had to be given back their pre-test papers it was not possible in the time available to record all individual errors, but only Sub-test and Total test scores. Similarly it was not possible to follow up individual errors with a diagnostic interview and error analysis.

Also in the time available it was not possible to consider undertaking any kind of remedial treatment program, other than the do-it-yourself business of returning corrected pre-test papers, providing the booklets, and offering any extra help that individuals might feel to be necessary. Nor was it considered ethically acceptable, in this Orientation situation, to test some students and not return their pre-test papers, or, in fact, to treat any group of the students differently from the rest. For this reason it was not possible to have any kind of Treatment or Control Groups.

These are all clear limitations on the study. However, the results are interesting, and stand out clearly despite the experimental limitations.

Results of the UPNG PY Basic Maths Study

		Mean	Stan. Dev.	Minim.	Maxim
Total	Pre	62.4	20.9	17.1	97.1
	Post	84.8	12.2	42.9	100
Fractions	Pre	66.1	26.6	7.1	100
	Post	89.2	12.8	35.7	100
Decimals	Pre	62.7	23.2	0	100
	Post	84.7	19.0	16.7	100
Directed Numbers	Pre	66.5	27.1	0	100
	Post	86.5	18.0	16.7	100
Order/Square & square Roots	Pre	54.4	21.2	11.1	100
	Post	75.0	17.9	33.3	100
Difference Between Total Post & Pre		+22.4	+17.3	-9.0	+74.3

TABLE 1: Percentage Results For Pre-Test and Post-Test on Total Test Score, Subtest Scores, and the Difference Between Total Score For Post-Test and Pre-TestA

Table 1 presents the results for Pre-Test and Post-Test Totals and Sub-Total Pre-Test and Post-Test scores expressed as a percentage. Because of clerical errors in compiling the scores for the subtests of Order of Operations and of Squares and Square Roots it has been necessary to lump these Subtest results together. However subtest results are only given out of interest, and do not form a major part of the analysis. Indeed all the Post-test mean scores are large improvements on the Pre-Test scores. The high Post-test scores are in fact precisely the kind of mastery level scores that we would hope to achieve on work that has been taught and retaught extensively for four years. Also the Standard Deviations of all the Post-Test scores are noticeably smaller than those of the Pre-Test scores. That is, the Post-Test scores are more tightly grouped around their corresponding means.

For the Total scores, as well as all the Sub-test scores, the differences between Pre-Test and Post-Test are all highly significant on a t-test on paired scores. That is, the increase between Pre-Test to Post-Test is not due to random effects. There is thus a clear statistical significance in the claim that the students really have improved thear test performance during the Orientation period.

The correlation coefficient for Pre-Test Total and the Difference Between Post-Test and Pre-Test Total is -.812. This means that the worse that students did on the Pre-Test the more likely they were to make large improvements on the Post-Test Total score, whereas students who did very well on the Pre-Test were likely to make only small improvements in their Post-Test. In fact some students who did well on the Pre-Test actually lost a few marks on the Post-Test, probably due to a combination of luck, or bad luck, and care-less errors. (Unfortunately, because the test papers were returned to the students it was not possible to make a further clinical analysis of "carelessness" in errors.) These Post-Test drops explain the negative Minimum score for the Difference between Post and Pre Test.

This relation between Pre-Test Total and Post-Test Total can be seen for the bottom twenty students in Table 2.

Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Pre	6	7	9	9	10	11	11	12	12	12	12	13	13	13	13	13	13	13	14	14
Post	17	33	27	25	30	23	28	15	27	23	30	30	28	32	19	15	32	33	28	32

Table 2: Raw Scores (n/35) on Pre-Test Total in Rank Order and Post-Test Total (Pre-Test Raw Mean is 21.9; Post-Test Raw Mean is 29.7)

In most cases the improvement between Pre-Test and Post-Test is enormous. However four cases stand out; students number 1, 8, 15 and 16. All of these make only a small improvement. With scores on the Post-test of about 50 per cent they are performing well below the intended mastery level. They comprise the tiny hard core in desperate need of remediation. But, more likely, given their probable general academic weakness, these four students will probably find that the work in PY is too hard. They will either drop-out or be counselled out, or fail ignominiously. They are students who have probably been away from school too long and need some Grade 10 brushing up, perhaps by taking Grade 10 courses with the College of External Studies, or by taking a more liesurely part-time Adult Matriculation course with the UPNG Department of Extension Studies.

Sullivan's study (1983, p9, Table 3) also found that, out of a large group that performed poorly on the Pre-Test, only two or three students showed virtually no improvement on the Post-Test. The remedial hard core usually turns out to be very small, despite initial appearances to the contrary on Pre-Test results.

Sullivan's study contained no girls. The PY study had a small number of girls, 11 out of a total of 96. The numbers were too small to provide substantial statistical analysis. However some descriptive statistics may be of interest.

Percentage Mean

		Male	Female	Total
Total	Pre	62	66.5	62.4
	Post	84	91.4	84.8

TABLE 3: Comparison Between Male and Female on Pre-Test and Post-Test Total Percentage Means

It appears that, with this small proportion of girls, the girls' Pre-Test marks are slightly higher than the boys' marks, though they are distributed uniformly through out the range of boys' marks. However their Post-Test marks are considerably higher than are the boys' marks. This is interesting, though such a small proportion of girls makes it difficult to draw firm conclusions.

Discussion and Conclusion

In this study of the beginning of Preliminary Year Mathematics students on basic skills screening tests the results are quite clear. Although many students at first seemed very weak, with virtually no assistance, nearly all students rapidly improved to an acceptable mastery level. Why did this happen? What might have happened instead?

Sullivan (1983, p7) suggests in his study that his "pre-test administered soon after the long holidays underestimated the students' performance". This post-holidays phenomenon is well known. In the case of the PY students, most had been away from school and formal mathematics for at least one year, and often several years. Such a long "holiday" could easily explain whole-sale forgetting of what may not have been well understood in the first place. However, no elaborate remediation program was implemented. Nor was there time for one. Fortunately there was little need for one.

Perhaps there is nothing that so effectively concentrates students' minds as telling them that the next test counts, and then giving them some good material to help. Of course, this study was concerned with basic mathematics that students should have already learned and practised for years. It is not, or should not be a matter for large-scale remediation programs but for small-scale on-the-spot reminding, once the small number of really weak students has been identified.

More generally, whenever mathematics courses begin by assuming familiarity with material, and then proceed to reteach for several weeks what may or may not be familiar, some judicious pre-test/post-test "reminding" may be a more effective way to deal with revision. There is little enough time for teaching new mathematics, without having to spend large slabs of time reteaching old stuff, with the likelihood of boring students who already know most of it. Sullivan's suggestions (1983, p13) for trade and vocational mathematics may be adapted to the more general situation:

> [pre-requisite] mathematics should not be [revised] as a separate subject, but where it arises in the [later application] courses... so that applications are given priority and the skills [are] taught in this context ... each course should establish a procedure for the identification of individual students weaknesses, and provide assistance where necessary.

What Sullivan found in this small-scale study has been observed in this larger PY study, and has still wider application to other kinds and levels of mathematics teaching. If you know they've already "done it", but they don't seem able to do it now, try a little kick in the pants - rub their noses in a diagnostic pre-test and tell them the next one will count.

3. VIEWING SCHOOL MATHS FROM A UNIVERSITY PERSPECTIVE

CAN RIGOUR BE INTRODUCED INTO SCHOOL MATHEMATICS? HOW EARLY?

KATHLEEN COLLARD
MATHEMATICS DEPARTMENT
PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

1. GENERAL

My position must first be made clear. I am looking at School Mathematics from the effect it has on University entrants who have to take mathematics as an important element of their courses. The University is sandwiched between school and the world of professional practioners of one sort or another. The employers of those practioners want to find in them men or women who are able to think about problems and worry out the solutions for themselves: people who are able to take initiative, to guide, and to solve the problems of those working under them. These graduates will have no one to turn to when they come up against difficulties themselves. In order to do all that it is required they must be people with trained minds who are able to think for themselves: much of their problem solving will entail converting a real life situation into the language of mathematics, and having the insight to see through to what mathematics is needed - it may be any branch from operations research to differential equations, or geometry, or even some moderately abstract area of pure mathematics. The metamorphisis of the student to such a state must be made in the time from when, as a school student, highly dependent on his teachers, brought up in a system of ten years at least of rote learning, reaches us, to when he/she graduates four or five years later.

Clearly the easiest thing for us to do is to continue the rote learning, but that is not going to provide the leaders that the country is hoping will come out of its Universities, and what the employers want.

I hope that what I say in the following will convince some people at least that the problem is not insoluble, and the schooling a child gets can help solve the problem.

Firstly it must be recognised that the required change cannot take place with the situation as it is at present. I will speak only about mathematical attainment of students, but what I say may well apply to other areas of education. Quite considerable amounts of new mathematics have to be studied by a student at University in order for him to understand the central subject of the course he is doing, whether it is a branch of engineering, chemistry, surveying, accountancy, to name just four. The amount of new work is in fact many times greater than the total of what is done in all of grades 1 to 12 with very much less teaching time available for it because of the competing demands of other subjects. At the same time, the failures of the student to cope with quite large areas of mathematics which he should have mastered at school should be corrected, and the attitudes of the student to his work altered. We try to cover these last two objectives as we introduce new work, but I think many lecturers at Unitech would agree that, to take just one example, the insecurity students feel in working with rational and negative numbers and manipulating rational algebraic expressions is so deep-seated it would take considerable concentrated effort to eradicate it. Add to that the more basic problem which, I think it is fair to say, is almost universal amongst students, and that is their failure to understand the basic processes of mathematics, including both their inability to comprehend a logical progress from one step to another in a logical argument and their inability to proceed from specific cases to general cases and vica versa. In plain words, they do not know how to "prove" anything (although we use the phrase "show that" rather than "prove", which appears to be a rather frightening word) and they cannot generalise. True they use the symbol ∴ but they often have no idea what it means and some invariably change it to =. For example this is the type of thing found in a student's work:-

Problem. Solve

$$\begin{array}{lll} & 3x + \frac{3}{4} & = x + \frac{7}{8} \\ = & 3x + \frac{3}{4} - x & = x + \frac{7}{8} - x \\ = & 2x + \frac{3}{4} - \frac{3}{4} & = \frac{7}{8} - \frac{3}{4} \\ = & 2x + & = \frac{1}{8} \\ = & x & = \frac{\frac{1}{8}}{2} \\ & & = \frac{2}{8} = \frac{1}{4} \end{array}$$

[Showing an inability to deal with rational numbers]

Finally, unless he is prompted, no check of the answer obtained is made in the original equation. If it is, either the student marvellously "fiddles" the numbers so that it does check - or if he finds that it does not, he usually leaves the whole thing in the air. He seems not to be able to think "I must have made a mistake, I wonder where it is".

Many of these ills stem from the fact that the majority of students have no idea of the nature of mathematics, they have no idea that it is an "architectural" subject, that is, it is built from the ground floor. Then you develop the second floor depending on and using what is on the ground-floor, and so on. The way elementary mathematics is developed at school leaves them with the idea that the "play mathematics" they do - adding piles of oranges, buying bilum, etc - are examples of mathematics, not as it should be, that these are real world situations and that mathematics is an abstraction from them. That idea is of course not easy and certainly should not be pushed down children's throats, but it could trickle into what the children do. For example, let the children have one pile of 3 oranges and one of 4 oranges (or any other objects of the same kind, which are available and familiar to them) and find out how many there are altogether. When they have done some similar exercises with other familiar objects they could move to the same sort of problem with less familiar objects and finally to a statement of the form

$$3 + 4 = 7$$

A large amount of such work is done - or should be-at Community School level. In my view the written material for Provincial High School should take into account to a greater extent what is done in the six years at Community School, and particularly so,as material gets written for Community School use,and efforts are made to improve the quality of teachers. A child with no great mathematical aptitude will go no faster than he is pushed; a child with better understanding may well become bored and lose interest if he does not move on quickly enough, and particularly if his teacher does not encourage him to ask questions about the work because he, the teacher, is frightened that the question will be beyond his capabilities to answer. (I mention that, because I saw it happening in Nigerian Schools in the 1950's). For that reason it is important that he should be offered some challenges in the written text.

It is in fact these children with mathematics ability who should be making their way to Universities to study the Sciences, Engineering and Commerce, and should be the future leaders of the country in those areas. Their minds need to be trained to think and not only to learn. That fact leads me to my main contention, which is, that, in the moves away from the "old" mathematics (which began in the 1950's), one of the principal aims was that mathematics should be presented so that the type of child who until then was not able to "do" mathematics at school (and invariably boasted the fact when he became adults) should be able to "do" it. In these moves it was, I think, generally assumed that the child who was "good" at mathematics, would be good whatever the material, and whatever the programme. That may well be true in countries where there are well qualified teachers who are able to take students if necessary beyond the written material. A teacher who himself has a good idea of the nature of mathematics can guide the thinking of a child along the right channels. Where there are not good teachers, as I have said above, motivation must come from the written material.

In my reading of "Mathematics Our Way", (which I consider has many good qualities and I would be the last person to belittle the work done in the planning for and writing of the books) I think there is a trend that way. I consider there could be more for the mathematically able child which would lead him on to an understanding of the nature of mathematics. In this country where classes are invariably large and often the teacher is educated only just beyond the level he has to teach it is important that the text books should stretch out to these more able children. There will certainly be arguments - we cannot have streaming - what if the less able children don't understand it? I feel sure that those difficulties can be overcome.

In M.O.W. grade IX there are "Enrichment Sections" - an idea which could well be developed and introduced in earlier grades. There is a teacher's guide to help the teacher, so he, if interested, might gain as well.

In case I be accused of pontificating let me give some suggestion as to how my ideas might be put into practice, here are some:

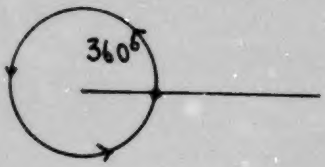
In Mathematics Our Way (M.O.W.) 7A.
Chapter 1., add these questions at the end:-

Activity 6. (Enrichment)

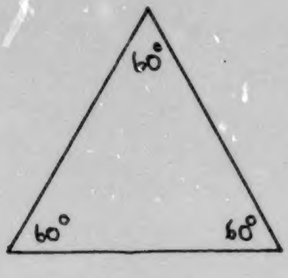
1. In your list in question 3 of Activity 4, (p. 7) you should have named three shapes which could make a tiling pattern, and two which could not, the pentagon and octagon. Why do pentagons not make a tiling pattern? Why do octagons not make a tiling pattern?

   (Hint - try to fit some pentagons together so that they all have a corner at one point. Will they fit right round the point without leaving a gap)

2. This angle, going right round a point is 360°.



   You are told that the angles of triangle you made are all 60°.



   Work out, using arithmetic, how many triangles will fit round a point.

   Does your answer agree with what you discovered in your tiling pattern?

3. (Use what you are told in question (2) about the angle going right round a point).

You are told that the size of each angle of your pentagon is 108°. Using arithmetic, can you explain why you could not get an exact number of pentagons to fit round a point.

4. (Harder).

(a) Can you explain why each angle of an octagon cannot be 120° (Use what you discovered in trying to fit octagons in a tiling pattern.

(b) From your tiling patterns discover which shape has all its angles 120°. (You can use the pattern on page 3 or on page 5).

(c) Find a tiling pattern from which you can find each angle of an octagon. What is the angle?

2. MATHEMATICAL IDEAS INTRODUCED THROUGH ARITHMETIC

In the "old" pre - 1950 mathematics children who were able to understand the work got an excellent idea of logical development and the structure of mathematics from a study of Euclid. In this study, which lasted over several school years, if I remember correctly, school children were:-

(1) told what they could assume,

(2) used what they were told to prove logically - i.e. showing that they were using only what they were told, to prove the first theorem,

(3) used only what they were told and what they had proved in the first theorem to prove the second theorem, and so on, for something like fifty theorems.

After proving a theorem, riders, that is problems, in which the theorem could be used were done. The problem usually consisted of drawing a figure and the child had to pick out in the figure where exactly the theorem could be used. (Often these riders were set for homework, leading to hard-pressed parents!)

I think it is fair to say that probably about 60% - 70% of any age-group (or more) could not make sense of what was going on. Even Euclid admitted this by calling theorem 5 "Pons Assinorum" - the bridge which asses have difficulty in crossing.

Difficult though it was, it should be noted that, certainly up to about 1940, possibly a few years later, in order to be accepted into a British University a candidate had to perform well above average in that Mathematics or equally well in Latin.

When the world war ended and nations began to recover from their wounds, considerable attention was paid to developing new teaching programmes, including those in mathematics. It was considered that,in the Computer age,which was imminent, more people would have to be able to think mathematically. It was also considered that many of the more recent developments in mathematics should enter the school programmes.

In some of the programmes there were aims at providing short chains of proofs, but in many a large amount of the work was of a descriptive nature and left the type of child who would, in the earlier days, have enjoyed doing Euclid, little to chew on. Certainly in English Universities there were indications that entrants who had done these courses were less able to follow logical arguments than had been the case earlier. There could of course be other reasons for that including the very great post-war expansion in British Universities, although entrants 'who' had done 'old' programmes at school seemed to stand out from their contemporaries.

It is my view that if Papua New Guinea wishes to localise its Mathematics teaching in Universities and other Colleges of Higher Education in the forsee-able future, the people who are to fill the positions must now have appropriate training in mathematics certainly in Provincial High Schools and as soon as possible in Community Schools. That work must surely begin by what is presented in Texts, and must prepare the child for the work he is to do in National High School and at the University.

In fact Arithmetic is a perfect medium for providing the logical development needed. The rules, which are drawn from observation of the physical world, are few. For the positive integers they are:-

(1) There is a first positive integer - called one (1).

(2) Each integer has another following it.

(3) There are two binary operations,

    (i) addition (+), giving the sum of two integers as an integer.

    (ii) multiplication (×) giving the product of two integers as an integer.

(4) Addition and Multiplication are both

    (i) Commutative i.e. $m + n = n + m$, $m \times n = n \times m$

    (ii) Associative i.e. $l + (m + n) = (l + m) + n$
    $l \times (m \times n) = (l \times m) \times n$

    (iv) Multiplication is distributive over addition
    $l \times (m + n) = (l \times m) + (l \times n)$

(5) One × any other integer = the integer back again,
One + any number = the next number.

N.B.
[I am not suggesting they should be included in any school text like that]

Examples of indiginous number systems that begin with one (not zero) abound. Interest can be found in how the numbers follow each other and exploring ideas of whether there is a largest number.

My view is the number line showing the ordered set of positive integers could be given to advantage early in grade 7 and number facts shown on the number line.

Rules 4(i) and (ii) seem fairly obvious to children when they are working with small numbers, less than 10. Is not that the reason why they became laws of mathematics? But are they so obvious when they are dealing with larger numbers, for example in adding two digit numbers (the way it is done on p.31 M.O.W. 7A), is not 27 + 48, really

(20 + 7) + (40 + 8)
= (20 + 40) + (7 + 8) [Commutative Law and Associative Law]
= 60 + 15
= 60 + (10 + 5)
= 70 + 5 (Associative Law)
= 75 ?

Seems a long string, and I would not propose that it should form part of the main text. What is done in the text and the practice following is to some extent needed. Some rote learning is needed. But the gifted child needs to understand why he does certain things and needs challenges beyond what rote learning provides. This could come in by enrichment, where problems in which there is considerable advantage in using the laws - problems that are fun - could be set. (I would think there are few who would deny that the majority of those set on pp.35 and 36 qu.4 and 5 would as well be done by hand calculator).

Such a process should continue throughout the four years of secondary school.

When the rules of subtraction and division are introduced (N.B. these are not new basic operations like addition and multiplication, but inverse operations to those) the number system has to be enlarged, of course. At the same time we have to add three more laws:-

(6) There is an integer 0 such that 0 + any positive integer = the positive integer back again.

(7) Every positive integer n has an additive inverse -n such that $n + {}^{-}n = 0$. -n is called a negative integer.

(8) Every integer except zero has a multiplicative inverse $\frac{1}{n}$ such that $n \times \frac{1}{n} = 1$.

Adding the new numbers, would mean of course renumbering the laws to make it clear that laws (3), (4) and (5) and (6) apply to all numbers. Also numbers (7) and (8) would probably be reversed, with slight changes in statements when fractions are introduced before negative numbers.

With the full range of real numbers and a proper understanding of - and + as inverse operations, an intelligent child should be able to get a thorough understanding of why the rules of arithmetic are as they are:-
for instance why (negative) × (negative) = positive; why dividing by $\frac{p}{q}$ is the same as multiplying by $\frac{q}{p}$, why adding ${}^{-}n$ is the same as subtracting n. With all these numbers there is plenty of scope for the child himself to explore. The previous mentioned facts are just some examples of what University students do not know with certainty and have no idea how to derive them. Many of these students fail in their later work in Algebra and Analysis just for that reason, and they seem unwilling to go back and re-learn the work again.

There are innumerable opportunities for enrichment, in M.O.W. to bring in ideas of proof. Very often, at the end of an Activity, enrichment,including some rigour,can be obtained by just asking the question "why".

Two examples are given in the following appendices which show that Mathematics has laws and how to use them.

APPENDIX 1

[After Question 5 of Exercise I. p. 32. M.O.W. 7A.Language will need to be adjusted to the capabilities of the children].

Enrichment

You will probably have noticed when you began to learn arithmetic that when you added two or three numbers this is what happened.

(a) In adding two numbers it did not matter the order in which you wrote them.

For example
$2 + 3 = 5$ and $3 + 2 = 5$.

(b) In adding three numbers, you first add two of the numbers and then add the third to the answer. But you got the same answer which-ever two you add first.

For example if you have to
find $1 + 3 + 5$ you could
either (i) add 1 to 3 to give 4 and then add 5 to that to give 9.
or (ii) add 3 to 5 first to give 8 and then add 1 to the answer to give 9.
or (iii) add 1 to 5 first to give 6 and then add 3 to the answer to give 9.

We can write these three ways using brackets, ( ), to show what you do first, like this:-

(i) $(1 + 3) + 5 \quad 9$,
(ii) $1 + (3 + 5) = 9$,
(iii) $(1 + 5) + 3 = 9$.

Because the answer is the same in all cases it is usual to leave the brackets out and write

$1 + 3 + 5 = 9$.

What is described in (a) and in (b) seems obvious when you are dealing with small numbers, and you may wonder why we talk about it. However the same results are true for all the numbers you will ever meet. Because of that Mathematicians have given names to the two properties.

Property (a) is called the Commutative property of addition. We would write (a) as

$2 + 3 = 3 + 2$ [Commutative Property]

(b) Is called the Associative property of addition. We would write (b) as

$(1 + 3) + 5 = 1 + (3 + 5) = (1 + 5) + 3$

using brackets, ( ) to show which two numbers you add first.

Question 1

(a) Find out if, when you add 21 and 35, the commutative property, holds: that is, find out if

$$\begin{array}{r} 21 \\ +\ 35 \\ \hline \end{array} \quad \text{and} \quad \begin{array}{r} 35 \\ +\ 21 \\ \hline \end{array}$$

give the same answer.

(b) Choose any two numbers, both larger than 10. Do you get the same answer for both of the ways in which you can write them down to add them?

(c) Choose any two numbers both larger than 100. Do you get the same answer whichever way you add them?

(d) Do $\begin{array}{r} 159 \\ +\ \underline{\ \ 67} \end{array}$ and $\begin{array}{r} 67 \\ +\ \underline{159} \end{array}$ give the same answer?

Question 2

In this question brackets are used to show which two numbers you add first. For example in finding (2 + 7) + 8 you would first add 2 to 7 to give 9 and then add 8 to the answer to give 17. We would write that

$$(2 + 7) + 8 = 9 + 8 = 17.$$

Write these additions in the same way

(a) Add 7 to 8 and then add 2 to the answer.

(b) Add 2 to 8 and then add 7 to the answer.

(c) What do you notice about the final answers of the example and the two additions you did in (a) and (b)?

Question 3

Do the following additions:-

(i) (27 + 6) + 3,
(ii) 27 + (6 + 3),
(iii) (27 + 3) + 6,

What do you notice about the answers?
Did you find any of the ways of adding these numbers a little bit easier than the others?

Question 4

Do you think the associative law applies to larger numbers? Try adding the numbers 81, 67 and 19 in three different ways.

Before you begin, look carefully at the numbers and see if there is one way of adding them which is going to be easier than the other two ways.

Question 5

Example (1)

We can write the addition 62 + 83 in this way

$$\begin{aligned} 62 + 83 &= 60 + 2 + 80 + 3 \\ &= 60 + 80 + 2 + 3 \\ &= (60 + 80) + (2 + 3) \end{aligned}$$

(because we have found the order in which we add numbers, and which two we take together first doesn't matter)

$$\begin{aligned} &= 140 + 5 \\ &= 145. \end{aligned}$$

Example 6

We can write 27 + 48 in this way

$$\begin{aligned} 27 + 48 &= 20 + 7 + 40 + 8 \\ &= 20 + 40 + 7 + 8 \\ &= 60 + 15 \\ &= 75 \quad \text{(straight away)} \\ \text{or} \quad &= 60 + 10 + 5 \\ &= 70 + 5 \\ &= 75. \end{aligned}$$

[That is what you do when you "carry" 1 into the tens place (p. 31)]

Write these additions (from p. 33) in the same way:-

(i) 47 + 82

(ii) 49 + 96

Example

To add numbers which have "hundreds" as well as tens we can do the same sort of thing and apply the rules we have learnt, to the numbers. For example

$$\begin{array}{rcl} 263 & = & 200 + 63 \\ & = & 200 + 60 + 3 \end{array}$$

You can quickly get used to doing that without having to write down all the steps.

Example

Add 263 and 647

$$\begin{array}{rcrcrcrcl} 263 & = & 200 & + & 60 & + & 3 & & \\ +\;647 & & +\;600 & + & 40 & + & 7 & & \\ \hline & = & +\;800 & + & 100 & + & 10 & = & \underline{910} \end{array}$$

Do these calculations in the same way:

(i) 359 + 427
(ii) 785 + 65
(iii) 48 + 463

APPENDIX II

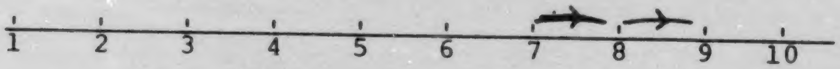
Enrichment [Before "Read This:" on p. 43 M.O.W. 7A]

When you subtract one number from another you are doing the opposite to adding two numbers. Look at this:-

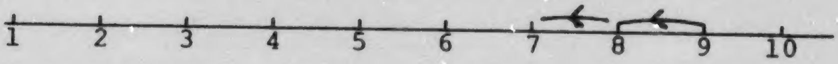
$$\begin{array}{r} 7 \\ +\ \underline{2} \\ =\ 9 \end{array} \qquad \begin{array}{r} 9 \\ -\ \underline{2} \\ =\ 7 \end{array}$$

You must add 2 to 7 to make 9. So when you take 2 from 9 you are left with 7.

You can also see how this works on the number line:-



7 + 2 = 9



9 - 2 = 7

The same relationship is true for adding and subtracting all pairs of numbers.

For example,

$$\begin{array}{r} 36 \\ +\ 17 \\ \hline 53 \end{array} \qquad \begin{array}{r} 53 \\ -\ 17 \\ \hline 36 \end{array}$$

Exercise

(1) Fill in the missing numbers in the following additions and subtractions:-

(i) $\begin{array}{r} 13 \\ +\ \underline{15} \\ [\quad] \end{array}$ $\qquad$ $\begin{array}{r} [\quad] \\ -\ \underline{15} \\ 13 \end{array}$

(ii) $\begin{array}{r} 121 \\ +\ \underline{136} \\ [\quad\ ] \end{array}$ $\qquad$ $\begin{array}{r} [\quad\ ] \\ -\ \underline{136} \\ 121 \end{array}$

(iii) $\begin{array}{r} 47 \\ -\ \underline{32} \\ [\quad] \end{array}$ $\qquad$ $\begin{array}{r} [\quad] \\ +\ \underline{32} \\ 47 \end{array}$

(2) When you are asked to do a subtraction like this one:-

$$\begin{array}{r} 67 \\ -\ \underline{35} \\ 32 \end{array}$$

do you see that it is the same as being asked the question "What number must be added to 35 to make 67?" The answer is clearly 32. Because:-

from $\begin{array}{r} 67 \\ -\ \underline{35} \\ 32 \end{array}$ $\qquad$ it follows $\begin{array}{r} 32 \\ +\ \underline{35} \\ 67 \end{array}$

Try to answer these questions.

(i) You need 78 toea to buy a (what-ever might be bought). You have saved up 55 toea. How much more must you save to have enough?

(ii) You need 100 coconuts for a party (mumu). You have got 67. How many more will you need to find? When you are trying to find the coconuts your uncle brings you 50. You will now have more than you need. How many more?

Or similar problems related to what they might do.

NOTE:
For both of the above type of problems students could be encouraged to devise their own problems. The emphasis should be on problems with large numbers. Students are willing to accept these laws when applied to small numbers, but they should see they apply to all numbers. The use of calculators would facilitate this practice. After all at this point of the work we are not focussing on learning number bonds. In any case with the brighter children, the assumption can be made that they know the number bonds and do not require extra practise.

IS THERE A PLACE FOR THIS KIND OF THING ....?

BASIL V. LANDAU
MATHEMATICS DEPARTMENT
PAPUA NEW GUINEA UNIVERSITY OF TECHNOLOGY

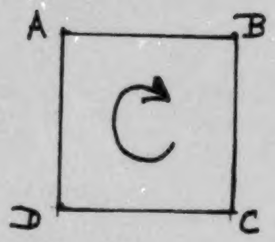
> "The feet of a table are in one plane, as they should be. But the floor is, alas, not plane.
> "Can we place the table so that all four feet are on the floor?"

I'm not suggesting putting topics like this on the syllabus - only that their discussion can widen horizons on what mathematics is about; also that links with more conventional syllabus material may be possible.

"But", I can already hear the chorus (perhaps with a qualification "Maybe in advanced countries, but not in Papua New Guinea"), "......We've not enough time as it is. And if we had more, there are more important things to spend it on".
I wonder.

* * * * * * * * * *

At one level one could experiment, with a desk in good condition on a bumpy part of the floor, or by using egg-boxes for floors and model animals for tables (but remember they represent good tables). But I would be disinclined to introduce the topic until I felt that at least some of the class could grasp the following argument, for a square table with feet A, B, C, D.



Suppose initially B and D are both touching the floor, but neither, or only one, of A and C are touching it. Turn the table, keeping two diagonally opposite feet on the floor all the time, until A and C occupy the positions originally occupied by B and D.
(We assume the table rigid and not so big, or so near to a wall that the walls would interfere with the turning process. We also assume that the floor, while possibly fairly bumpy, is not jagged, and consequently the turning can be carried out smoothly. Now back to the argument). Since B and D are initially on the floor, and A and C on the floor later, there must be a change over moment, or maybe a more extended period, when both pairs of feet are on the floor.

* * * * * * * * * * *

A student who grasps this argument, and to whom it was not previously obvious that a position could be found with all feet on the floor, has felt the power of mathematical argument. She or he has experienced proof as turning something external - accepted, if at all, on flimsy experimental evidence, or on a teacher's authority - into something of his own.

Of course this is proof at a very informal level. As it should be. The assumptions in parentheses, (and others not stated here) should probably not be mentioned first time round. They are intended as merely making explicit what most students would accept intuitively as inherent in the terms of the problem - ideally they should be stated in response to a student's objection or to help a student overcome a difficulty in grasping the argument.

Often associated with proof are rote-learning, and a selection by authority of what must be accepted as obvious (so that e.g. congruent triangles are used in proving the fact, at least as obvious to most as any about congruent triangles, that opposite sides of a rectangle are equal). These may have made "proof" a dirty word for some educationists; but they are, it is hoped, absent here.

Continuity plays a key role in the argument, but at the level of a child who knows that in reaching the top of a hill he will at some stage be half-way up. At a certain level one could make a link here with better-known class-room material, by introducing an "angle of rock" which is, say, postive when the table can rock about BD and negative when it can rock about AC. (Mathematically, though not physically, it would be better to imagine B and D kept on the floor throughout, negative angle of rock corresponding to A or C piercing the floor. In fact for small departures from flatness the two measures are almost the same). If we plot angle of rock against angle of turn, the smoothness of the turning process is reflected in the angle of rock having to go through zero in getting from positive to negative.

This brings me to the point that introducing extra-curricular material is a double-edged affair. On the one hand we ought usually to link it to more ordinary mathematics so that the new topic does not appear too isolated from the main stream of students' work - they already compartmentalise their knowledge too much. On the other hand there are dangers that the link will, for some students, take the fun out of the new topic, and that, for others, the link may cause confusion when earlier they thought they were understanding the new topic - so why cloud the issue by bringing in negative numbers?

And this is where the chorus of "Not enough time" may in a sense be right - not because there are more important things than giving students the insight and maybe, excitement or fun, that can be achieved in this way, but rather because adequate preparation by the teacher to avoid various pitfalls may be so demanding.

Nevertheless, I remain an unrepentant advocate of the view that

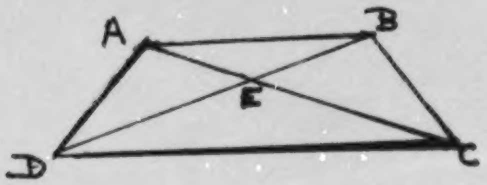
> a syllabus should contain a clause saying that so many hours should be (or, at the least, that up to so many hours may be) devoted to topics not on the syllabus

Some of my compatriots might call such a document an Irish syllabus, but I prefer the analogy with a Soviet 5-year plan which, so I once heard, stipulated for scientific institutions that 10%, or whatever, of their work, should not form part of the plan.

* * * * * * * * * * *

At whatever level the theme of Good Tables on Bad Floors is presented, questions can easily be asked which cannot be properly answered without going to a higher level. For example a class that can just manage the argument for a square may want to generalise: does it work for a rectangular table, for any quadrilateral, for a table with five feet?

As the rectangle seems to me, in spite of greater symmetry, no easier than the isosceles trapezium, I will discuss the latter shape, unusual as it may be for a table. (The application described at the end of this paper was to just this case, though, to be fair, the angles were not far off right angles).



Suppose then that the feet A, B, C, D form a figure with AB//DC and AD = BC, and that, as in the treatment of the square case, B and D are initially touching the floor. (The argument can, of course, be adapted to the case where instead A and C touch the floor).

We may use a similar argument to that for the square, but this time we specify that during the rotation, not only is there a pair of diagonally opposite feet on the floor at any time, but also that the intersection E of the diagonals stays on a fixed vertical line. (We might have introduced such a stipulation for the sake of definiteness in the square case - it may even assist the imagination rather than cramping it). The lack of square symmetry means, however, that there is now no guarantee that we can get A and C to occupy the positions initially occupied by B and D. For though A and C are the right distance apart from each other and from E for this, the new position of B and D might force the level of E above the initial one, preventing A and C simultaneously touching the floor. However this objection falls if E is initially is at the highest point, H say, attainable by E on the fixed vertical during a complete rotation of the table. By choosing an initial position of the table such such that E and H coincide, instead of permitting any initial position, as can be done in the square case, the argument that there must be an intermediate position with all four feet on the floor becomes available for the isosceles trapezium.

A proof on the above lines will obviously be considerably more sophisticated than the one for the square. One should not, however, over-estimate the degree of sophistication, which lies not, as some might suppose, in the ideas used, but in the way they hang together to form an argument. Thus, though we use the equality of AE and BE, which we did not have to use in the square case, this occasions no difficulty, being evident from symmetry. And the existence of a highest position H for E as the table is rotated will be intuitively clear to most students. Some would deny this, or question the propriety of its acceptance in a proof, on the ground that the theorem about a continuous function attaining its upper bound is not a trivial one, but has some subtleties in its proof. But I would say that this shows that the formalised, arithmetised, notion of continuity does not capture exactly its geometrical precursor, rather than showing the untrustworthiness of the latter. (Or, to those who would challenge this, that sufficient unto the day is the rigour thereof).

Much easier is the question whether there are any plane configurations of four feet for which we can't guarantee the existence of a position with all feet on the floor. Any plane non-cyclic quadrilateral will settle this - if the floor shape is part of a large sphere it is easy to show there can be no position with all feet on it.

And cyclic quadrilaterals other than those already discussed? These present a problem of a greater order of difficulty altogether. It is known that if the floor's departure from flatness is small, then the minimum rocking angle is, if not actually zero, then very small indeed. This tends to suggest that the answer is yes. However, so far as the writer knows, it is still one of the many problems in mathematics which are easy to state but have not been solved.

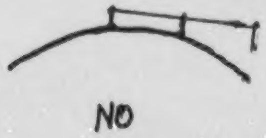
* * * * * * * * * * *

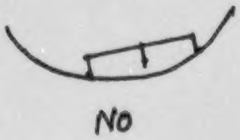
It is good if students are stimulated to ask their own questions. As the treatment of the square table is fairly simple, the questions may well be, as has been suggested, in the direction of generalising the problem. But, as the distinguished Polish mathematician warned his young compatriot

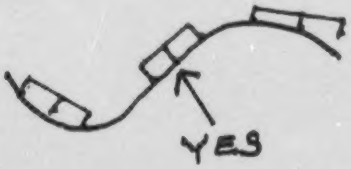
Mark Kac, subsequently to become an American mathematician of note, "you must earn the right to generalise". A way to begin, which often brings new insights whether or not it eventually earns you this right, is to try to generalise not in the usual sense (generalising upwards), but in the direction of simplicity (generalising downwards) or even of triviality. (In spite of the fact that "trivial" was originally a contemptuous reference to courses of study with no mathematics, what might be called "the art of the trivial" plays a valuable part in mathematics).

In our table problem what can be simpler than the square case, other than the 3-legged stool which can obviously be placed with all feet on the floor? To answer this we may imagine ourselves living in a kind of "flat-land", say a vertical plane, where the floor, instead of being a curved surface is just a curve, on which it is obviously possible to place a two-legged "table" anywhere (provided it is small enough and not too near the "wall").

What about a "good" 3-legged table? Good here means that its feet are in a straight line. The answer in general is no, but it depends on the curve.







There is now no turning, only moving along, but notice that in the "Yes" case illustrated the required position is again transitional between two positions of different types where fewer than the full number of feet touch the floor. Also in the first "No" case the table can be rocked about the central foot. And the "angle of rock" (in conjunction with the dimensions of the table) measures the local departure of the floor from straightness.

This will not work in the second "No" case of course - unless we have a very special type of 3-legged table whose outer legs can pierce the floor! But the same measure of local departure from straightness can be reached by considering the distance of the central foot from the curve (again in conjunction with the dimensions of the table). In fact there exist instruments for measuring plane curvature, with an adjustable central leg, based on just this idea.

This brings us back to the land of ordinary floors and tables, because the instrument is better known in its 3 - dimensional version, the spherometer, for measuring the spherical curvature of a surface. Here three feet form an equilateral triangle, while the fourth central leg is adjustable. For a convex floor, spherical curvature might alternatively be measured by the rocking of a "table" of this shape, but with the central foot in the same plane as the equilateral triangle.

But more important is the fact that while for a curve the local departure from straightness is characterised by a single quantity, the departure of a surface from flatness requires two. One can be taken as the spherical curvature. The other is a measure of departure from sphericity. And now we are back to our square table, because the local departure from sphericity is connected with the maximum rocking angle when we rotate a small square table on that part of the surface. For plane and spherical surfaces there will be no rocking.

The journey to flatland and back has helped to point to a possible application of table turning which has nothing to do with the original problem.

* * * * * * * * * * *

Talking of applications, one of the attractions, to some people, of the problem this paper began with, is its "practical" flavour. It could be claimed that this practicality, as in many other "applied" problems, is spurious. For what we really wish to do is to stop the table rocking while keeping its sides parallel to the walls - so the traditional procedure of wedging, not turning, is the right one.

Against this, I had the following experience soon after I came to Papua New Guinea. My bathroom here has an irregular floor, shaped so that overflow from sink or shower can drain away. Having to replace a light bulb I took a chair to stand on, but it rocked. I knew that it did not rock on the lounge floor so its feet formed a plane figure, and by its symmetry that this figure was an isosceles trapezium. I therefore turned the chair in complete confidence that I would find a position where it did not rock. And thus indeed it proved, and I was able to mount the chair in safety.

4. POST YEAR 10 MATHEMATICS TEACHING

THE QUALITY OF THE GRADE 10 SCHOOL LEAVERS POOL

LAWRENCE ROS

In this paper I will discuss:

1. The effect on standards in National HIgh Schools of increasing significantly the number of students admitted to Grade 11.
2. The effect such an increase will have on standards at the tertiary level.

In the discussion I will mainly use data from my own research project: MONITORING STANDARDS IN MATHEMATICS OF THE GRADE 10 OUTPUT.

INTRODUCTION

It is often argued that: "An immediate expansion of National High Schools would improve the quality of education simpy by the fact that two more years of schooling will be provided for more students. Two more years would not hurt anyone, specifically for the number of students that would be granted places. Most western and developed countries provide twelve years of schooling for their children. Why not therefore bring Papua New Guinea to equivalent standards."

Or one might say: "Expanding National High Schools is the means to supply adequately 'entrants of quality' into the tertiary institutions".

Others may be more careful and argue that: "A proposal for increasing or doubling the number of National High Schools i.e. doubling the number of places available to students, would seem to lower standards unless there was an equivalent increase or doubling of Provincial High Schools since 1978, when the last of the four National High Schools opened. A slower National High School expension would seem to be more in line with the growing pool of Provincial High School students suitable of National High School entrance.

A fact is that such arguments are not very often substantiated by hard evidence. During an argument one would say; Clearly, the number and the quality from the Provincial HIgh School are there!" or "It is axiomatic that there will be quality in that pool" or "It can not be argued that there are too few of the proper quality in that pool"

THE SCHOOL LEAVERS POOL

We will first consider the 1983 pool of school leavers (ex Grade 10) In 1983 there were 7663 students enrolled in Grade 10 and 824 of these were selected for studies at the four National High Schools. We will of course concentrate on achievements in Mathematics as measured by the D_2 test. (For details such as validity etc. See my 1984 report or other papers).

It has been standard practice in the Department of Education to rate achievements in terms of Distinction (5%), Credit (20%), Upper Pass (25%), Pass (40%) and Failure (10%).

On the basis of these ratings the following table was obtained, and added are the Mean and S.D. of scores in the D_2 test of students who obtained these ratings.

TABLE 1

	DISTINCTION	CREDIT	UPPER PASS	PASS	FAILURE
N	383	1532	1915	3065	766
MEAN	33	24	18	11	10
S.D	7	6	6	4	
TOTAL IN THE 4 N.H.S.S	255	430	38	-	-

NOTE: I do not always obtain information of all students. The last row should therefore be read as: at least.

In Table 2 similar data are listed for the 1981 National High School intake. In that year 847 students had been selected out of a pool of 6128 Grade 10 leavers.

TABLE 2

	DISTINCTION	CREDIT	UPPER PASS	PASS	FAILURE
N	306	1225	1531	2450	613
MEAN	31	24	19	12	10
S.D	7	7	6	4	-
TOTAL IN THE 4 N.H.S.S	171	424	121	9	-

THE QUALITY OF THE STUDENTS IN THE POOL.

The following data are taken from my paper 'Transition Rates and Mathematics Achievement in Five Developing Countries' (1980). They refer to the results of the numerical ability test D_2: In the graphs students' score is plotted against the number of students achieving a given score. The D_2 test is a mastery test and contains 50 questions covering material that should have been mastered by Grade 8 students, and certainly by students who had entered the post-Grade 10 level. Only slips of the pen were allowed.

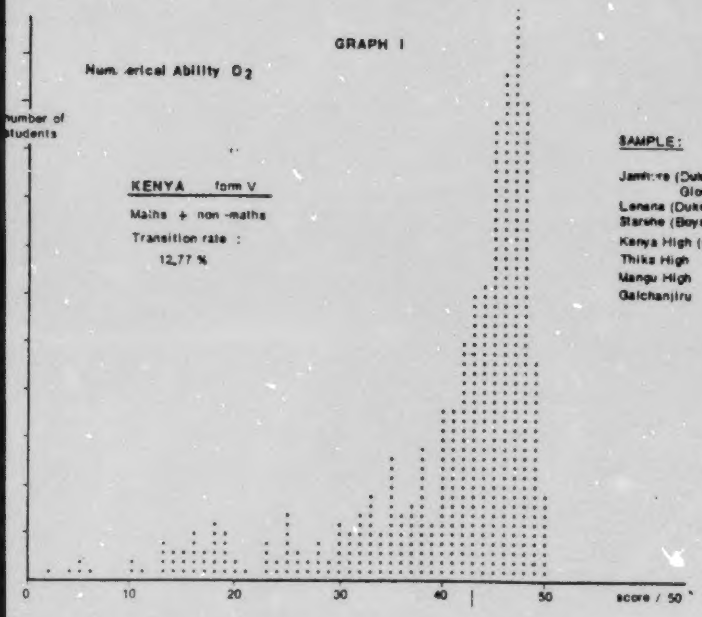
A vertical line was drawn at the 43/50 (86%) score. Students scoring higher than 86% are considered to have sufficient potential for tertiary studies involving mathematics.

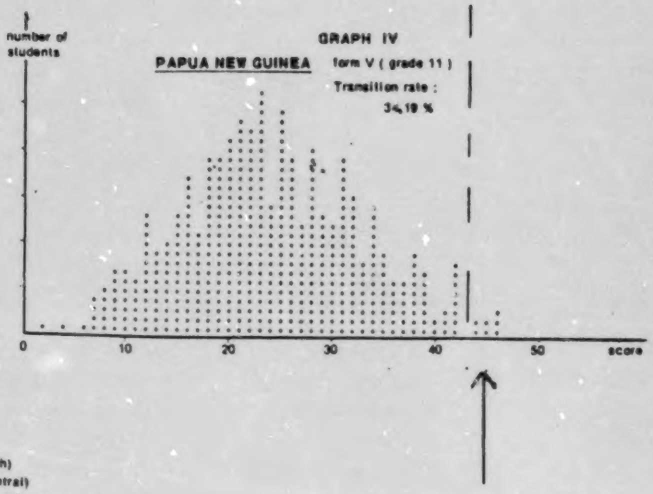
It should be pointed out that for each country the graphs represent the results of the total grade 11 sample i.e. of all students irrespective of whether they took mathematics or not.

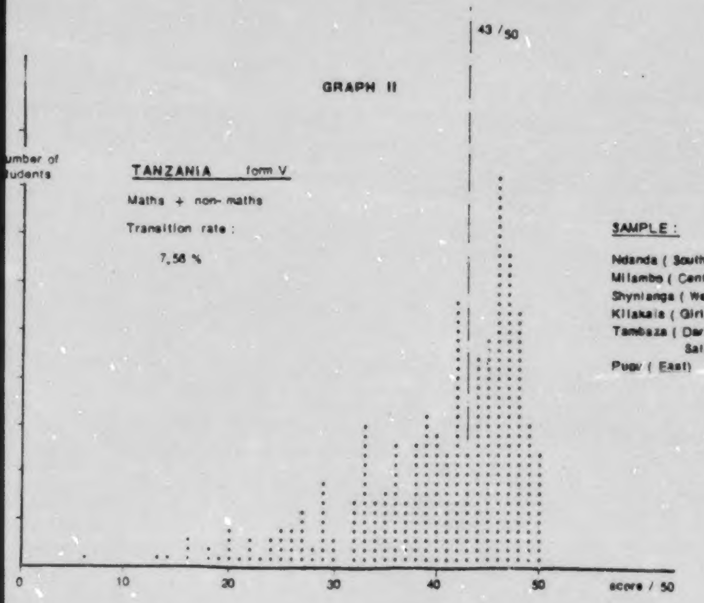
From the graph it can be seen that in PNG only 7 out of 459 students achieve mastery.

Out of a total intake of 797 (1979) this would be 12 or 1.5%. For Kenya this would be 304/510 or 60% and for Tanzania 192/369 or 52%.

One has to recognise that Papua New Guinea has like all societies its share of students of high intelligence. However what distinguishes PNG from say Kenyan and Tanzanian education systems is the fact that the educational attainment of its students has been severely compromised by its poor formation they have received in the Primary and Secondary schooling.







Before considering the implications of this some further points must be made about the quality of the school leavers pool.

The table below shows the quality of the intake of the Primary Teachers' Colleges.

TABLE 3

MATHS	DISTINCTION	CREDIT	UPPER PASS	PASS
1980	11	157	323	527
1983	4	206	411	360
1984	39	315	452	225

The Mean for Teachers' College and Technical College students in the D_2 test is in the order of 12 - 14 with S.D of 6 e.g. for students of the Lae Technical College in 1982 the Mean was 12.30 and S.D. 6.12.

The 1979 - 1980 Report of the Office of Higher Education Central Selection Unit (more recent reports could not be obtained) shows a.o. that 26% and 25% of respectively UPNG and GTC Preliminary Year students did not meet stated academic criteria.

For the 1980 approved enrolment of 1066 students at Primary Techers' Colleges 87% of the students did not meet stated academic criteria of a minimum of 3 Upper Passes.

Given this, we are now in a position to establish what sthe effect of increasing the number of National High Schools on standards in the schools will have.

TRANSITION RATES AND STANDARDS

The research data are summarised in Table IX and XII of the 1984 report which is available from the author.

The Data show conclusively that:

1^0 Standards in Mathematics of the Grade 11 intake as measured measured by the D_2 test are low and have been low since the research started in 1974.

2^0 Lowering the transition rate (Grade 10/11) from 14.9 to 10.8 over the period 1978 - 1984 has resulted in a significant improvement of the quality of the Grade 11 intake.

The correlation between Transition Rates (Grade 10/11) and achievements in terms of D_2 results is -0.912.

DISCUSSION

At a number of occasions in the past concern has been expressed about the understanding of basic mathematical topics of the National High School graduates. Snell and Priest both of the Engineering Department of PNGUT voiced this concern during the annual Mathematics Research Seminar in Lae (1983) in the following terms:

"Many of our students at present exhibit only an ability to recite from memory or to produce or to produce a standard solution".

"The feeder institutions only teach and test the skill of recall".

From my experience as a member of the National High School Mathematics Syllabus Review Committee and as a member of the Examinations Committee and also as a lecturer in Mathematics of the Year One students at Goroka Teachers' College, I fully realise that the criticism is justified.

On the other hand it is without any doubt in my mind that National High School teachers in Mathematics go to the limit of their resources each year in order to bring a mixed batch of distinction, credit and upper pass students up to some standards, at present roughly equivalent to O-Level Cambridge.

It would be sad if the National High School component of the National Education System,- the only component where standards are comperable to international standards -, was pressured into a situation which would force standards down.

There could very well be a significant lowering of morals.

CONCLUSION

Increasing significantly the number of National High School places at this stage would result in admitting low-credit and upper pass students. (Students scoring less than 20 in the D_2 mastery test)

Increasing significantly the number of National High School students will logically increase the number of eligible students to proceed to Universities. This will however inevitably lead to a fall in standards of achievements unless the criteria for awarding matriculation are made more stringent.

A side effect of significantly increasing the number of students admitted to National High Schools can be derived from Table 3. The table shows a very substantial improvement in the quality of students being admitted to the 9 Primary Teachers" College in 1984. The number of admitted to the 9 Primary Teachers" College in 1984. The number of Passes since 1980 has been cut by 60% and the number of Credits has been doubled.

Increasing the number of National High School places will undoubtedly go at the expense of the quality of the Teachers' College intake. This too would be an unhealthy development.

APPENDIX I

The argument against increasing significantly the number of National High School places at this stage, would run along the following lines:

a. Even in 1984 National High Schools accepted Upper Passes and low Credits. Such students are under-achievers in the PNG education system.

b. Adding 200 of these under-achievers and mixing them with the other 800 will result in slower progress in the teaching, less effective teaching, and a lower attainment. National High School teachers informed me about the considerable difference it makes in teaching a group of students of mean 22 in D2 or of mean 26. This is backed up by my own experience.

c. Increasing the number by more than 200 at this stage would be an outright disaster and destroy the National High School system. Loading up a system with students of average ability, below average ability or with under-achievers will always go at the expense of retarding the development of the more gifted.

d. The effect of high transition rates on standards in schools is well illustrated by the experience in Provincial High Schools where over a period of 15 years, and largely as a consequence of high transition rates Grade 6/7, three different mathematics syllabi, each progressively less academically demanding than the former, have been introduced, trialled and with the exception of the present (a modified version of the third of these), abandoned.

Before considering the effect of increasing significantly the number of students admitted to Grade 11 on standards at the Tertiary Level we must consider the quality of the Grade 12 output.

The following data are obtained from Report no. 25 of the Mathematics Education Centre on Grade 12 students' mathematical performance on the 1980 and 1981 external examinations.

The scores of students in both the major and minor papers are listed in the following table:

1980	major course	Mean	55.87%	S.D	16.75
	minor course	Mean	48.3%	S.D	13.7
1981	major course	Mean	44.8%	S.D	14.8
	minor course	Mean	47.7%	S.D	12.4

The author commented on the results in the following terms:

"As in 1980 the topic scores are low, far too low for these students' future teachers to have confidence that they had a good command of the subject matter in the year 12 syllabi."

In 1980 a total of 473 out of 776 Grade 12 leavers were granted either matriculation or provisional admission status.

(Approximately 61%)

It was shown above that widening entry to National High Schools would result in lowering standards or making it more difficult to raise standards.

Increasing the number of Grade 12 students would result in a proportional increase in the number of students who would gain matriculation.

Hence if the number of students admitted to Grade 11 would increase significantly even the bottom students presently completing Grade 12 and others with inferior Grade 10 results, will be designated matricula

This will lead inevitably to a fall in standards of achievement at the Tertiary Level.

DISTANCE VERSUS FACE-TO-FACE LEARNING: A MATHEMATICS TEST CASE

G.S. KAELEY

1. THE PROBLEM & ITS SETTING

A comparative Study of the Performance in Preliminary Year Mathematics of Mature-age Distance Learners, Mature-age Face-to-Face Students and High School Entrants.

1.1 The Statement of the Problem:

What effect do the following variables have on achievement of Preliminary Year Mathematics of distance learners, mature-age internal candidates and high school entrants of the University of Papua New Guinea in 1984?

(a) Mode of study.
(b) Number of years away from school and mathematical experiences gained during that period.
(c) Numerical ability.
(d) Attitudes toward Mathematics.
(e) Sex.

1.2 Theoritical Frame Work:

Discussing about acceleration theory (based on mastery learning) and enrichment theory (based on the normal curve of initial ability) of learning, Taylor and White (quoting Fredrick and Walbery 1980) write that

> "Enrichment theory is based on the assumption that little or nothing can be done to improve the level of achievement of individual students, and that the normal curve of achievement is a reflection of the normal curve of initial ability".[1]

---

[1] J.C. Taylor & V.J. White, "Student achievement as a function of perceived study time: a comparative analysis" in J.S. Daniel et al (eds) Learning at a Distance: A World Perspective, Athabasca University/ICCE, Edmonton, 1982, p. 130.

According to the advocates of this theory, differences in achievement are more or less fixed by differences in innate intelligence and are therefore not significantly altered by increased effort and study time. On the same lines it may be argued that method of learning i.e. face-to-face versus distance, will not make much difference in mathematics achievement. From this perspective numerical ability test and Grade 10 result in mathematics would be a strong predictor of achievement in Preliminary Year mathematics.

The time elapsed since leaving school and returning to study affects the learning of adults in a number of ways. Firstly the number of years away from school may weaken the habit of learning and of expecting to learn. Secondly an adult relearns most easily what he learned earlier but not too long ago.[1] Therefore, the duration of break in studies will affect the learning of mathematics of mature students. However, if after leaving school he has been working in mathematically oriented occupation, those mathematical skills are reinforced and will contribute to his achievement in mathematics positively on returning to study. This research will examine the validity of the enrichment theory.

Interests, attitudes. concepts and values become more fixed with age. Attitudes of older people are more stable. This very stability makes for the "resistance to change" and thus making the learning process, difficult. The attitudes of youth change more easily.[2] Therefore, the achievement in Preliminary Year mathematics will be dependent on the students' attitudes toward mathematics.

---

[1] J.C. Zahn, "Differences between Adults and Youth affecting learning" in F.A. Karnes et al (ed), Issues and trends in Adult Basic Education, Focus on Reading, University Press of Mississippi, Jackson, 1980, p. 29.

[2] Ibid, p. 24

Zahn drawing on Goldfarb research on 18 to 65 year old people writes that the physical reaction time of men is quicker than that of women but the speed of reaction slowed with age.[1] From the last many centuries our society continues to expect from men to adopt more mathematically oriented jobs and to know more mathematics than women. Therefore, the attainment of male students is expected to be higher in Preliminary Year Mathematics than that of the female students. The application of enrichment theory to sex differences in mathematics will also be studied.

1.3 Hypotheses:

(1) Comparable mature-age distance learners achieve results similar to those of the mature-age internal students in Preliminary Year Mathematics.

(2) The achievement in Preliminary Year Mathematics is inversely proportional to the number of years away from school both for distance and internal mature-age learners.

(3) The mature-age internal and distance students whose occupations were mathematically oriented achieve higher grades in Preliminary Year Mathematics than those of the similar non mathematically oriented occupation entrants.

(4) The attainment in Preliminary Year Mathematics is directly proportional to the students numerical ability and their Grade 10 Mathematics result.

---

[1] *Ibid*, p. 31

(5) *The achievement in Preliminary Year Mathematics is directly proportional to the students' attitudes toward Mathematics.*

(6) *The level of attainment of male students in Preliminary Year Mathematics is higher than that of their female counterparts.*

1.4 Definitions:

School leaver: A school-leaver in this study will mean a student who will join the university in Preliminary Year (PY) straight after completing the 10th grade. He will be abbreviated as SL.

Non-School leaver: A non-school leaver is a mature-age university entrant, who had spent at least one year either working or staying at home after completing 10th grade but before securing admission as an internal student in PY. Abbreviation for him will be NSL.

Distance learner: A distance learner is a mature-age student, who is at least 21 years of age and has a minimum of two years work experience before enrolling for Matriculation Studies (MS) which is the external equivalent of PY. He has been learning through a combination of correspondence courses and a limited amount of face-to-face teaching. He will be abbreviated as DL in this study.

1.5 The importance of the study

The high dropout and failure rate and low achievement of students in general and NSLs in particular at the University of Papua New Guinea (UPNG) and the Papua New Guinea University of Technology (PNGUT) in all Mathematics courses is a cause of concern to university authorities as well as to the teachers.[1] There is very little research done on the comparative study of the performance of NSLs and SLs. Some of the research papers written were for faculty submissions. For example Pernetta, Robert and Mcwbray have studied the performance of Grade 12 entrants and ex-PY students in the foundation year and have submitted their results as papers to the Science Faculty. Murphy has done a similar study on the foundation year students and has published her paper in the Papua New Guinea Journal of Education.[2] Their findings, however, do not show comparison of SLs and NSLs. Kember conducted research on the performance of DLs at UPNG[3] and Kaeley evaluated the Matriculation Studies Programme.[4] These two papers discussed DLs only. Kaeley has also written on the performance of foundation year NSLs in Mathematics,[5] which has not shown comparison with any of the other groups of students. Aside from the last not much research has been conducted specifically on the teaching of Mathematics at UPNG. There is hardly any published work on the comparative performance of NSLs and SLs.

---

[1] P. Clarkson & H. Whippy write in "Papua New Guinea University Student Attitudes to Mathematics", in P. Clarkson (Ed.), Research in Mathematics Education in Papua New Guinea 1981, Mathematics Education Centre PNGUT, Lae, p. 22, that the mature age students seem to be early casualities of the tertiary system; some dropping out with their quiet departure unnoted by teaching personnel until weeks after they have gone. They have also acquired a reputation for earning poor results if they do stay on. Why this situation prevails no one seems to know. In view of their special usefulness, they need to be investigated.

[2] P.S. Murphy, "National High School versus Preliminary Year Studies: Performance of UPNG Foundation Year Students 1979" in Papua New Guinea Journal of Education, Vol. 17, No. 1, Port Moresby, 1983, pp. 82-91.

[3] David Kember, "Some FActors affecting Attrition and Performance in a Distance Education Course at the University of Papua New Guinea" in Distance Education, Vol. 2, No. 2, Open University, Milton Keynes, 1981, pp. 164-188.

[4] G.S. Kaeley, Distance Education at the University of Papua New Guinea, an evaluation report on Matriculation Studies, Department of Extension Studies, University of Papua New Guinea, Waigani, 1980.

[5] G.S. Kaeley, "Teaching Mathematics at First Year Level to Mature Students, at the University of Papua New Guinea", (mimeographed), Department of Extension studies, University of Papua New Guinea, 1982.

At PNGUT, the Mathematics Education Centre (MEC) and the Department of Mathematics have carried out a number of research projects on the teaching of Mathematics. They have some research papers on the comparative performance in Mathematics of different groups of students in the foundation year. Jones and Wilkins indicate that the performance of foundation year NSLs is so low as compared to other groups, that they should not be admitted to the course before being asked to complete the PY Mathematics.[1] Clarkson and Whippy have similar findings in their research on attitude and Mathematics performance.[2] The research evidence existing at PNGUT, points to low performance in Mathematics of the NSLs. There is no study even at PNGUT comparing the achievement of NSLs and DLs.

From the above discussion it is clear that no comparative study exists on the performance of NSLs, SLs and DLs in Mathematics in Papua New Guinea (PNG). The achievement of NSLs in PY and foundation year Mathematics is worse than in other subjects both at UPNG and PNGUT. As in other countries, Mathematics appears to be regarded as a difficult subject by many students in PNG as well.[3]

Distance education at university level has been well researched. Quoting Mackenzie et al (1977), Giampaolo Bonani writes that although correspondence education, T.V. and radio have also been used for qualification or diplomas at the secondary level, but research at this level is incomplete and sporadic.[1] There is not much study of this nature in other countries of the world. Therefore, this research on pre-university mathematics teaching in PNG will have universal relevance.

---

[1] P.L. Jones, and C.W. Wilkins, "Mathematics Achievement at Tertiary Level: A Comparative Study at PNGUT", in Papua New Guinea Journal of Education, Vol. 17, No. 1, Port Moresby, 1983, p. 77.

[2] P. Clarkson and H. Whippy, "Papua New Guinea University Student Attitudes to Mathematics", in P. Clarkson (Ed.), Research in Mathematics Education in Papua New Guinea 1981, ..., pp 21-35.

[3] N. McIntosh and V. Morrison, "Student Demand, Progress and Withdrawal-an analysis of the Open University's first four years" in Higher Education Review, Autumn, 1974, write that many mathematicians would assert that Mathematics is simply inherently more difficult. A.J. Bishop, "Implications of research for mathematics teacher education" in Journal of Education for Teaching, Vol. 8, No. 2, Methuen & Co. Ltd., 1982, p. 121 writes, quoting Duchworth and Entwistle (1974) using a Kelly type repertory grid technique modified for a group approach, showing that school children do not only perceive mathematics as a difficult subject but also as an important subject to study.

Hence, it is important to undertake this study. The knowledge of factors which contribute to the performance in Mathematics of different groups of students might help to improve their attainment. It may reveal why students perform in a certain way, why do they perform in that way, and who perform better in Mathematics than the others. The knowledge of these factors may:

- save students from wasting their time;
- allow the university to prepare them better;
- provide high schools with feed back;
- save sponsors' money;
- improve standards of Mathematics and hence of science and other related subjects in the country;
- suggest reasons for the alleged low Mathematics standards in PNG and give remedies for it;
- suggest a better method of teaching Mathematics to NSLs and DLs;
- provide insight into the problems of SLs, NSLs and DLs;

It is for these reasons, as well as other insights that may be obtained in the course of the research, that this study is being undertaken.

---

[1] Giampaolo Bonani, Institute Teologico per Corrispondenza, Italy, "Correspondence Teaching: Second choice or Second Class"? in John S. Daniel et al, Learning at a Distance: A World Perspective, Athabasca University/ICCE, Edmonton, 1982, p. 19.

2. THE REVIEW OF THE RELATED LITERATURE

2.1 A Historical Overview:

In the earlier times mature persons were the main recipients of education. A continuing education was the method of instruction. The family, clan and tribe were responsible to impart necessary education to the growing up individuals so as to make them fit for life. Later on more specialized type of education started to be given through a method of apprenticeship. Thus in the olden times education or skills training was mainly taught to the mature people. Then started the organised institutions for teaching adults. The examples of such institutions are the ancient universities of Texiela and Nalanda in Asia and the medieval universities in Europe, where education was for the mature persons or part-time learners.[1]

The word 'education' is of recent coinage, although learning centres existed much earlier. Education of children was first mentioned in France in a document of 1498. In the English language the word education first appeared in 1530 and it entered languages of other western countries later on.[2] It later became a monopoly of the children.

The relative importance of part-time course provision within the three main divisions of higher education, the university sector and the further education sector, has been consistently different in Britain. The medieval English and Scottish universities began as residential institutions for full-time students and this has remained a dominant pattern throughout the development of the university system. The growth of the non-university institutions of higher learning during the 19th century was, however, based on their part-time provision, and the emphasis has remained evident in the 20th century[1].

---

[1] J.R. Kidd, in the Foreword to Ignacy Waniewicz, Demand for Part-time Learning, Ontario, Educational Communications Authority, 1976.

[2] Ivan Illich, "The Alternative to Schooling" in A.G.M. Ishumi, T.L. Maliyamkono (eds), Education and Social Change, Black Star Agencies, Dares Salaam, 1980, pp. 9-10.

There was a concern to provide educational opportunities to those who were left out by the traditional system in Britain. One of the solutions to help such persons was the creation of the University of London in 1836 as a body to conduct examinations and award degrees. By 1858 the University started admitting matriculated candidates for its examinations from any part of the world. The mature students were helped to prepare for the examinations by the voluntary bodies such as the Workers Educational Association, the Extension Services and the Extra-mural departments of universities. It is parallel to many other countries, especially to the Soviet Union, where today nearly half the students in higher education study on a part-time basis.[2]

The British pattern of adult education was repeated in the U.S.A. and in the commonwealth countries, especially Canada, Australia and New Zealand. Derek Whitelock quoting Fred Alexander (Adult Education in Australia 1959) writes

> "... adult education in Australia since the Second World War, not withstanding the self sacrifice of devoted teachers and voluntary workers and a record of some substantial achievement, is still suffering from the effects of a system which was imported lock, stock and barrel from the United Kingdom nearly half a century before"[3]

---

[1] Malcolm Tight, Part-time Degree Level Study in the United Kingdom, The Advisory Council for Adult and Continuing Education, Leicester, 1982, p. 3.

[2] Norman Mackenzie et al, Open Learning, the Unesco Press Paris, 1975, p. 15.

[3] Derek Whitelock, "A Brief History of Adult Education in Australia and an outline of current provision", in Derek Whitelock (ed), Adult Education in Australia, Pergamon Press, Australia (Rushcutters Bay, NSW 2011), 1970, p. 1.

However, Wilson writes that University extension on English model was developed slightly earlier in America than Australia. Australia borrowed ideas both from Britain and America.[1] He further writes that the Evening Continuation schools established under Education Acts of the 1880s and 1890s on partly British and partly American experience were never successful. It was not until the 1940s and drawing largely on later American experience, that the New South Wales Education Department superseded these with its new form, the Evening College, and demonstrated effective means of meeting these needs.[2] The universities in Australia were quite slow to offer education facilities to the adults up to recent times. Up to 1970 only six of the 16 universities in Australia have departments of adult education. Universities in the U.S.A. and Canada do much more to provide education to the mature persons.[3]

University education in PNG was available to the adults through the University of Queensland before the establishment of UPNG. The University of Queensland Act 1909 made a provision that university education must be placed within reach of persons who are unable to attend lectures. Therefore, the University started external studies in Queensland in 1911 with extra-mural and correspondence courses, which were extended to other Australian states, PNG, Northern Territory, South East Asia and the Pacific islands later on.[4] Quoting Tertiary Education in Australia (1964) Volume 1, Dunton writes that the Queensland University has 404 students in New Guinea, comprising of teachers, members of the public service and from other occupations.[5]

---

[1] Lascelles Wilson, "Australian Adult Education compared with Overseas Systems" in Derek Whitelock (ed), Adult Education in Australia ..., p. 233.

[2] Ibid, p. 234.

[3] A.J.A. Nelson,: "The Universities and Adult Education" in Derek Whitelock (ed), Adult Education in Australia ..., p. 51.

[4] A.F. Dunton "The Australian Experiment in Teaching External Students at Universities" in Derek Whitelock (ed), Adult Education in Australia ..., p. 188.

[5] Ibid, p. 191.

According to David Sewart there are examples of the use of written material for educational purposes almost from the beginnings of written records. Therefore, what is new is not teaching at a distance, it is but the growth and popularity of distance teaching institutions.[1] The wider use of distance education began in the industrial cities of 19th century, England, Germany, Scandinavia and the U.S. It is mainly in Australia and Canada that it does belie its urban origins and embrace vast distances.[2] It is since the beginning of 1960s that it has gained popularity in the developed and developing countries alike. For example between 1960 to 1975 over 20 distance teaching institutes were set-up in Africa. Between 1972 and 1980 the number of correspondence institutions in Australia listed in the Commonwealth directory grew from 15 to 46.[3]

Nineteen hundred sixties brought a general upsurge of distance education and 1970s started the period of establishment of the open universities, both in developed and developing countries. The Open University in the U.K. enrolled students in 1971. It was followed quickly by new foundations in Spain (1973), Pakistan (1975), Athabasca in Canada as an Open University (1975), Hagen in Germany (1975), Israel (1976), Iran (1978), Sri Lanka (1980) and in Netherlands (waiting funding).[4]

---

[1] David Sewart, "Individualizing Support Services" in John S. Danniel et al (eds), Learning at a Distance - A World Perspective, Athabasca University/ICCE, Edmonton, 1982, p. 27.

[2] Desmond J. Keegan, "From New Delhi to Vancouver: Trends in Distance Education" in John S. Danniel et al (eds), Learning at a Distance - A World Perspective ..., p. 42

[3] H. Perraton, "Distance Teaching North and South" in John S. Danniel et al (eds), Learning at a Distance - A World Perspective ..., p. 16.

[4] Desmond J. Keegan, "From New Delhi to Vancouver: Trends in Distance Education" in John S. Danniel et al (eds), Learning at Distance - A World Perspective ..., p. 42

The phenomenan of older people entering higher education, once the exclusive right of the young, is both recent and widespread. It has been a movement of the seventies and has occured in most western countries.[1] Since early seventies, many Australian tertiary institutions started special entry schemes for mature-age students. La Trobe University in 1972, was probably the first to initiate special provisions for unmatriculated mature-age entrants.[2] In 1978 37 per cent of all enrolments in Australian Universities were part-time internal or external students. In colleges of advanced education 44 per cent were part-time students. In New Zealand, 38 per cent of all university students were part-timers.[3]

Canada has a large number of mature age students in the tertiary institutions. It has a population less than half of the U.K., but has almost as many students on full-time courses of higher education and many more part-timers. In 1978-79 over 36 per cent of University undergraduates and 42 per cent of postgraduates in Canada were part-timers, compared to proportions of under 2 per cent and 36 per cent respectively in the U.K.[4]

In the U.S.A. 41 per cent of the total enrolment in institutions of higher education in 1978 were part-time students.[5] Now the number of students who take part in programmes of further and continuing education has surpassed the number of traditional students.[6]

---

[1] Leo H.T. West *et al*, "Reasons for Entry into Higher Education" in T. Hore *et al* *Mature Age Students in Australian Higher Education*, Higher Education Advisory & Research Unit, Monash University, 1980, p. 86.

[2] Elena G. Eaton *et al*, "Academic Performance of Mature Age Students: A Recent Research in Australia" in T. Hore *et al*, *Mature Age Students in Australian Higher Education* ..., p. 48.

[3] Malcolm Tight, *op. cit*, pp. 74-76.

[4] *Ibid*, p. 74.

[5] *Ibid*, p. 76.

[6] Herbert Mainusch, "How can the Conventional University Serve the Distance Learners," in John S. Danniel *et al* (eds) *Learning at a Distance - A World Perspective* ..., p. 187.

The socialist countries such as the U.S.S.R., China, Korea, East Germany and Tanzania have developed their educational systems by integrating field experience with schooling for the mature-age entry people.[1] No doubt there are different mature-age entry schemes in operation in different universities in the U.K., but the Open University (OU) is mainly catering for the educational needs of the working adults. many Indian universities are providing direct entry to mature entrants either as full time or part time evening students or are offering distance study packages. Other developing countries in Asia, Africa, Latin America and the South Pacific have similar schemes, modified only to suit the local conditions.

The Papua New Guinea Experience:

Since its inception in 1965, UPNG has adopted various innovations to provide higher education facilities to the adults of PNG. Some opportunities have been offered to mature-age persons to enter the university as regular students. Lahara (Summer) Sessions have been conducted for them since 1971 in degree level credit and non-credit courses.[2] In 1977 the university started Arts Foundation Year (AFY) courses for working adults by combining Lahara and correspondence teaching.[3] In addition, the mature-age secondary school leavers have been given opportunities for matriculation studies, which served either as enrichment courses or to qualify for university admission. The following reasons may be given for offering such higher education facilities to the mature-age candidates in PNG:

Firstly, it is to give a chance for further studies to gifted adults in the society, who did not have opportunities for persuing higher education for some reason. Secondly, the national high schools (NHS) are not producing enough students to fill the places in the two national universities.[4]

---

[1] C.V. Mmari, "The Implementation of the Musoma Resolution. The University of Dar es Salaam Admission Experience" in A.G.M. Ishumi et al, Education and Social Change ..., pp. 168-171.

[2] Department of Extension Studies Handbook 1978, UPNG, p. 36.

[3] Ibid, p. 37.

[4] The Preliminary Year Courses were started to overcome this situation.

Thirdly, the situation in the country requires the production of high level man-power, in the shortest possible time, to localise the positions held by expatriates both in the public and private sectors.

3. THE RESEARCH DESIGN

According to Kerlinger[1], a research design is, in a manner of speaking, a set of instructions to the investigator to gather and analyze his data in certain ways. The data-gathering procedures suggested by Mouly[2] will be used in this research. His system would classify the data-gathering method proposed for this research as a self-report descriptive survey and the date-processing methods as statistical empiricism.

The methodology for this research project can be discussed under the following three headings.

3.1 The Problem and Theory:

The heading of this research is "Distance versus Face-to-Face Learning: a Mathematics Test-Case". It is a comparative study of internal PY University students especially NSLs and DLs. The members of both these samples are adult learners; therefore, it will have the application of theories of adult education and distance study. Malcolm Knowles' theory of adult education advocates that the adult learner comes to the class with a treasure of personal experiences. The teacher should recognise it and should try to make use of it to promote self directed learning. His emphasis on self-directed learning has bearing on distance education as well. On the other hand Knox suggests that the main aim of education should be to inculcate the spirit of life-long learning in the students.

Knowles' suggestions hold good for humanities and social sciences where actual life experiences help the mature students to understand these disciplines. In the case of mathematics and science the life experiences do not help much, unless the students were engaged in occupations involving the use of mathematical or scientific skills before returning to study. But even then the entrants who have not proceeded with their mathematics beyond the O level (Grade 11) had very little chance of applying their knowledge except in a routine capacity[1]. This implies that work experiences have less significance for the learning of mathematics at this level.

---

[1] Fred N. Kerlinger, Foundations of Behavioural Research, Holt, Rinehart and Winston, Inc., New York, 1967, p. 280.

[2] George J. Mouly, The Science of Educational Research, American Book Company, New York, 1963, pp. 300-301.

The distance education theories of autonomy and independence put forward by Charles A. Wedemeyer and Michael G. Moore have little relevance to the present study. These theories require that the learner should be independent and autonomous in choosing objectives, selecting resource persons, books, other media and pace of learning and method for evaluation[2]. The learners involved in this study do not have autonomy and independence for any of these factors, as they are controlled by the University authorities.

The second set of theories of interaction and communication in distance education is due to Borje Holmberg, John A. Baath and David Sewart. Holmberg believes that distance education is a guided didactic conversation[3]. It is suited to the needs of individual student who is more or less independent of the direct guidance of tutors. In the same way Baath considers that in its essence distance study is individual study[4]. He thinks that the desirability of telephone or face-to-face contacts can be calculated from the models for the various types of goals as presented, for instance, in Bloom's taxonomy of learning objectives[5]. On the other hand Sewart has sumed up his theory of distance education as continuity of concern for students learning at a distance. He argues that the study package can not be sufficient for the students, as each individual learner has different needs from the other. Therefore he suggests that face-to-face session is an essential component of distance teaching[6].

---

[1] Assistant Masters Association, The Teaching of Mathematics in Secondary Schools, Cambridge University Press, 1973, p. 25.

[2] Desmond Keegan, Six Distance Education Theorists, ZIFF, Hagen, 1983, pp. 7, 14.

[3] Ibid, p. 29.

[4] Ibid, p. 33.

[5] Ibid, pp. 33-34.

[6] Ibid, pp. 35-36.

The above mentioned theories have some relevance for the DLs of the present study, but the theory of industrialisation put forward by Otto Peters seems to be more suitable. He argues that the analysis of teaching at a distance in terms of conventional instructional theory has proved unsuccessful and unproductive and that one must therefore seek another basis of analysis. He proposed new categories for the analysis of distance education taken from economical and industrial theory. He argues that "whoever is professionally concerned with education today must acknowledge that there exist two clearly differentiated forms of teaching: traditional face-to-face teaching based on interpersonal communication and industrialised teaching based on technical and prefabricated forms of communication"[1] (1973:310).

In the industrial era of mass production of goods, education cannot remain interpersonal alone. Only the mass productive fashion of education can offer instruction to the individual learner with a freedom to choose his own way. Instruction is not linked to fixed times, to fixed places and to fixed persons. This assigns new responsibilities to the learner which were not characteristic of pre-industrialized systems. The various demands put on education today cannot be met by conventional methods, therefore, new techniques must be found, which must be industrial[2].

The two types of mature students of the present study have different needs. The NSLs can afford to forego their jobs and can manage to stay away from their families while the DLs wish to continue with their employment and living with their families. Their need for education cannot be met by conventional means. The alternative way open to them is through distance education, which is industrialised form of learning. According to Peter's theory various demands put on education in the present age can only be met by distance education.

---

[1] Ibid, p. 19.

[2] Ibid, p. 25.

3.2 The Method of Field Work and Data Gathering:

The method of field work and data gathering for this study is discussed under the following sub-headings:

(a) The Sample: The population of this study is the PY and MS students of UPNG. Among the 125 PY students, 106 entrants will be NLSs and 19 SLs. Out of the NSLs, the candidates coming either from overseas countries or from agricultural colleges or from previous year PY (i.e. repeaters) will be excluded from the sample. Their presence in the sample could bias the results of the research. This will leave about 80 NSLs and 19 SLs in the sample. A random sample of about 90 DLs will be taken from the University Centres at Arawa, Rabaul, Madang and Port Moresby. By using the above mentioned criteria, any overseas students and the entrants who studied full time in any institution in 1983 will be excluded.

The average female population of UPNG over the past years has been approximately 12 percent[1]. On this assumption, there will be a total of about 20 females in the whole PY and MS population. The overseas candidates and 1983 repeaters will be excluded, as in the earlier samples. This may leave a sample of about 12 females. To make up the size of the sample, female students from 1982 and 1983 will be included. A similar stratified sample of males will be taken to study the sex differences in the learning of mathematics.

The results of the pilot project of 1983 students indicate a high drop out rate both in NSLs and DLs. Out of 66 NSLs only 34 continued up to the end of course. Similarly from 120 DLs of Arawa, Rabaul and Port Moresby Centres only 70 students persisted till the final examination.

---

[1] P.S. Murphy, "UPNG as an agent of social stratification: an analysis of output by sex". A paper presented at the Extraordinary Meeting of the Faculty of Education, 1983, Waigani, p. 1.

Same type of situation is envisaged for the 1984 entrants. It means the above mentioned samples will be further discussed under two sub-samples i.e. of those who continue the studies for the whole year and of those who drop out on the way. These two sub-samples will be analysed separately and the results will be shown in the research report.

(b) The Instruments: The following instruments will be used in this study:

(i) Numerical Ability - D_2 Test: The Numerical Ability - D_2 Test used in this study is the one developed and used by Lawrence Ros of Goroka Teachers' College since 1974. It contains 50 questions covering material that should have been mastered by Grade 8 students, and certainly by students who had entered the post-Grade 10 level[1].

The predecessor of the D_2 Test called D_1 test was developed by him in 1974. It was designed and validated to give a mean of 25 and a standard deviation of 10 for PNG students at the Grade 11 level. It was slightly modified in 1976 and called D_2 test after algebra had been excluded from the Provincial High School syllabus. The test includes number facts, place values, order of magnitude, patterns, ratio and proportional parts, and tests of confidence in working with numbers. The results of D_2 test were correlated with:

(i) the results of the Pacific Numerical Reasoning Test;
(ii) Grade 10 leavers results in mathematics; and
(iii) first semester mathematics results for UPNG students at the Waigani and Goroka campuses. The correlations were all very high[2].

---

[1] Lawrence Ros, "Transition Rates and mathematics Achievement in Five Developing Countries", Goroka Teachers College, Goroka, (mimeographed), undated, p. 3.

[2] Ibid, p. 4.

(ii) Attitudinal Scales 1: Attitudinal scales 1 for this study have been developed by modifying the scales developed and used by MEC, PNGUT in 1982. The purpose of their scales was to find the attitudes of students toward mathematics especially on the appreciation of the subject, not necessarily linked to their final achievement[1]. While doing so they included statements mostly on the like/dislike of mathematics and anxiety about the learning of subject. For this study the spectrum has been broadened by deleting redundant and absolute statements and inserting relative and new statements. The statements are on anxiety of learning the subject, mathematics examination, interest, importance and utility of mathematics, perception of mathematics teachers, books, students own mathematical skills and sex. Wherever possible opposite statements have been introduced to check the validity of the students' responses. After every two statements on "Attitude toward Mathematics" a statement on "Sex and Mathematics" has been incorporated, to make the students think while completing the scales. All statements of the same type put together could be randomly ticked by the candidates, without showing their actual responses. The two examples about mathematics in the beginning of the scales could have biased the responses of the students. They have been replaced by neutral examples. Thus the instrument finally contains 18 statements on general Likert scale on "Attitude toward Mathematics", consisting of nine positive and nine negative statements concerning mathematics with five response columns. The other nine statements with five response columns are on "Sex and Mathematics", five of which are on equality of males and females in the learning of mathematics, two positive toward males and two positive toward females.

---

[1] P. Clarkson and H. Whippy "Papua New Guinea University Student Attitudes to Mathematics" in P. Clarkson (ed), Research in Mathematics Education in Papua New Guinea 1981, ..., p. 21.

(iii) *Attitudinal Scales II: The second attitudinal scales: "Attitudes to doing Mathematics" was also developed by the MEC, PNGUT. They submitted 24 statements to 70 second year students. The results were factor analysed and 10 statements on four factors of learning mathematics were used on the responses of 206 first year students. Only one of the statements was dropped*[1]*. The resulting nine statements form Attitudinal Scales II for this study.*

(iv) *MS Student Initial Information Questionnaire: A questionnaire is designed to find initial background information of DLs. It contains questions on students' personal, social and educational background.*

(v) *PY Student Initial Information Questionnaire: Similar to MS questionnaire but modified to suit internal students, the questionnaire is designed for the PY students as well.*

(vi) *Ms Student Final Information Questionnaire: A final questionnaire seeking the difficulties experienced in learning mathematics by the DLs will be developed.*

(vii) *PY Student Final Information Questionnaire: A final questionnaire similar to the MS questionnaire will be drawn up for the PY student as well.*

The questionnaires and attitudinal scales were pre-tested on the 1983 PY students and certain amendments were made in the light of their replies. The questionnaires and attitudinal scales consist to a large extent of closed questions. The strengths and shortcomings of closed questions are obvious. They produce quick response and are easy to process, but on the other hand, they tend to lead the interviewed person in a certain direction. The interviewed person may tick the one or the other alternative simply in order to please the interviewer or to get over with the questionnaire without revealing his real opinion on a particular item. As a last open alternative, the questionnaire provides for "other reasons" in most of the items so that the student could express his own views. In the case of PY and Ms students, since the majority of them are mature persons, one would assume that the students are able to critically analyse the alternatives and to tick the correct items according to their own opinions. However, one should also discount a certain amount of accuracy because the students may try to present themselves in the best possible way.

---

[1] *Ibid*, p. 28.

(b) Data Collection: In the first week of 1984 academic year all the students will be given Numerical Ability - D_2 test and Attitudinal Scales I. The administration of attitudinal scales in the beginning of the course will provide the entrants unbiased responses as the difficulties experienced in the learning of mathematics during the year may effect their attitudes toward mathematics. To find the background information of students, PY Student Initial Information and MS Student Initial Information questionnaires will be administered. To determine the difficulties experienced in the learning of mathematics by NSLs and DLs, PY Student Final Information and MS Student Final Information questionnaires will be given to them in the second half of the year. Attitudinal Scales II on students' attitudes to learning of mathematics will also be administered at the same time. The scores obtained by students in D_2 test, in 10th Grade mathematics and in PY and MS based on continued assessment and final examination will be sources of primary data. The published studies and texts and the unpublished dissertations and theses dealing with the performance of SLs, NSLs and DLs will be the sources of secondary data.

3.3 Method of Data Analysis:

As Paul D. Leedy[1] suggests research data can be classified under four categories i.e. historical, descriptive survey, statistical and experimental data. Each of these types of data requires a methodology suitable to its own nature. It is impossible, for example, to apply a methodology to data which in themselves are inappropriate to the demands of that method. For instance there is no way of getting any meanings from historical data by using experimental methodology. The data for this study will be collected by using questionnaires, and therefore according to him will require methodology of the descriptive survey method. The data analysing techniques suitable to this type are of descriptive statistics. The statistics used will reveal the points of central tendency, variation, and the degree of interrelationship between the variables in the data. The data for hypotheses 2, 4 and 5 especially are quatitative in nature, which requires methodology of the analytical survey method. The methods employed for this type will be those techniques whice are known as inferential statistics.

---

[1] Paul D. Leedy, Practical Research Planning and Design, Second Edition, MacMillan Publishing Co. Inc., New York, 1980, p. 76.

Some research has been conducted at UPNG using the statistical techniques mentioned above. For example David Kember[1] has studied, "Some factors affecting attrition and performance in a distance education course at the UPNG", by employing descriptive statistics method. He and David Mowbray[2] discussed performance of PY students compared to grade ten results by using both descriptive and inferential statistics. The complexity of the hypotheses to be tested in this study requires the use of both descriptive and inferential statistics. This study is similar to the research on Radio Mathematics in Nicaragua[3] which used both these types of statistics comprehensively to analyse the data.

In this study there are two types of variables: dependent and indepedent (explanatory). The dependent variable is achievement in mathematics. It will be based on assignments, tests, mid-year examination and final examination. The weighting given to assignments, tests and examination will be in the ratio of 3:2:5. In PY there will be 8 one hourly tests spread over the whole year. These tests will be combined to form 4 two hourly tests for the MS students, due to limitations on the availability of face-to-face time with them. It will put the latter group at a disadvantage in tests. It is presumed that the difference introduced in achievement due to it will not be significant. There will be a number of explanatory variables. The independent variables in hypotheses 2, 4 and 5 will be tested by using a mathematical model discussed below. Hypotheses 1, 3 and 6 will be tested by using other statistical methods. The variables used in this study can be sumed up as follows:

(a) Dependent Variable:

Achievement (ACH)
DEP (1) Achievement in mathematics of distance learners (ACHDL)
DEP (2) Achievement in mathematics of non-school leavers (ACHNSL)
DEP (3) Achievement in mathematics of school leavers (ACHSL)

---

[1] David Kember, op cit.

[2] David Mowbray and David Kember, "Performance of PY students compared to Grade Ten results", (mimeographed), UPNG, Waigani, 1981.

[3] Jamesine Friend et al (eds), Radio Mathematics in Nicaragua, Institute for Mathematical Studies in the Social Sciences, Stanford University, Stanford, California, 1980.

(b) Explanatory Variables:

(i) The explanatory variables for hypotheses 2, 4 and 5 are

EXP (1) Number of years away from school for distance learners (YRWDL)
EXP (2) Number of years away from school for non-school leavers (YRWNSL)
EXP (3) Numerical ability of distance learners (NADL)
EXP (4) Numerical ability of non-school leavers (NANSL)
EXP (5) Numerical ability of school leavers (NASL)
EXP (6) Grade 10 grade in mathematics of distance learners (GTGDL)
EXP (7) Grade 10 grade in mathematics of non-school leavers (GTGNSL)
EXP (8) GRade 10 grade in mathematics of school leavers (GTGSL)
EXP (9) Attitude of distance learners toward mathematics (ATTDL)
EXP(10) Attitude of non-school leavers toward mathematics (ATTNSL)
EXP(11) Attitude of school leavers toward mathematics (ATTSL)

The mathematical model to be tested for hypotheses 2, 4 and 5 will be represented by the following equations:

(1) ACHDL = f (YRWDL)
(2) ACHDL = f (NADL)
(3) ACHDL = f (GTGDL)
(4) ACHDL = f (ATTDL)

(5) ACHDL = f (YRWDL , NADL)
(6) ACHDL = f (YRWDL , GTGDL)
(7) ACHDL = f (YRWDL , ATTDL)
(8) ACHDL = f (NADL , GTGDL)
(9) ACHDL = f (NADL , ATTDL)
(10) ACHDL = f (GTGDL , ATTDL)

(11) ACHDL = f (YRWDL , NADL , GTGDL)
(12) ACHDL = f (YRWDL , NADL , ATTDL)
(13) ACHDL = f (NADL , GTGDL , ATTDL)

(14) ACHDL = f (YRWDL , NADL , GTGDL , ATTDL)

Similar 14 equations will be written by changing DL to NSL and 7 equations by changing DL by SL in equations (2), (3), (4), (8), (9), (10) and (13).

These 35 equations representing the mathematical model for hypotheses (2), (4) and (5) will be tested by using step by step multi-linear regression analysis. First the effect of one explanatory variable on dependent variable will be studied. Then the effect of two, three and four explanatory variables on dependent variable will be analysed.

(ii) The following statistical techniques will be used to study variables involved in hypotheses 1, 3 and 6.

Hypothesis 1: To study the comparative achievement of DLs and NSLs, the students from both the samples will be paired one to one by using the criteria of age, sex, work experience, type of jobs they did and Grade 10 result in Mathematics. It will help to control the effect of other intervening variables. Then the means of scores in mathematics of DLs and NSLs will be computed. If μ_d is the mean scores in mathematics of DLs and μ_n of NSLs, then the following two statistical hypotheses will be tested:

(1) $\mu_d - \mu_n = 0$

(2) $\mu_d > \mu_n$

The individual pairs will be studied to see if there is a significant difference between the scores of any of them. Then further explanation will be obtained from the data for any such discrepancies.

Hypothesis 3: Hypothesis 3 is involved with the relationship between achievement and mathematically oriented occupations. The students whose occupations were mathematically oriented will achieve higher grades in mathematics. The variable 'mathematically oriented' will be determined by the following factors:

- type of job a student has done in the case of NSL and has done or is doing now in case of DL. The occupations will be rank ordered and scores assigned to them.

- Weighting will be given to any mathematics studied in courses attended since leaving high school.

The total scores obtained by each student will be correlated with their achievement scores using Pearson's Correlation formula[1].

$$r = \frac{\epsilon x y}{\sqrt{\epsilon x^2 \quad \epsilon y^2}}$$

Hypothesis 6: Hypothesis 6 on achievement and sex will be studied by computing the means of scores in mathematics by male and female students. If μ_m is the mean in mathematics scores of males and μ_f of females, then the following two statistical hypotheses will be tested:

(1) $\mu_m - \mu_f = 0$

(2) $\mu_m > \mu_f$

To find the variance within the samples and between the samples, one way analysis of variance will be applied. These results will be interpreted in the light of the students responses to 'Attitude toward Sex and Mathematics'.

---

[1] Henry L. Alder _et al_, _Introduction to Probability and Statistics_, W.H. Freeman and Company, San Francisco, 1968, p. 187.

BIBLIOGRAPHY

1. Alder Henry L. *et al*, *Introduction to Probability and Statistics*, W.H. Freeman and Company, San Francisco, 1968.

2. Assistant Masters Association, *The Teaching of Mathematics in Secondary Schools*, Cambridge University Press, 1973.

3. Bonani Giampaol, Institute Teologico per Corrispondenza, Italy "Correspondence Teaching : Second Choice or Second Class"? in Daniel J.S. *et al*, *Learning at a Distance: A World Perspective*, Athabasca University/ICCE, Edmonton, 1982.

4. Clarkson P. *et al*, "Papua New Guinea University Student Attitudes to Mathematics" in Clarkson P. (ed), *Research in Mathematics Education in Papua New Guinea 1981*, Mathematics Education Centre, PNGUT, Lae.

5. *Department of Extension Studies Handbook 1978*, UPNG, Waigani.

6. Eaton Elana G. *et al*, "Academic Performance of Mature Age Students : A Recent Research in Australia" in Hore T. *et al*, *Mature Age Students in Australian Higher Education*, Higher Education Advisory & Research Unit, Monash University, 1980.

7. Friend Jamesine *et al* (eds), *Radio Mathematics in Nicaragua*, Institute for Mathematical Studies in the Social Sciences, Standford University, Standford, California, 1980.

8. Illich Ivan, "The Alternative to Schooling" in Ishumi A.G.M. *et al* (eds), *Education and Social Change*, Black Star Agencies, Dares Salaam, 1980.

9. Jones P.L. *et al*, "Mathematics Achievement at Tertiary level : A Comparative Study at PNGUT" in *Papua New Guinea Journal of Education*, Vol.17, No 1, Port Moresby, 1983.

10. Kaeley, G.S., *Distance Education at the University of Papua New Guinea*, an evaluation report on Matriculation Studies, Department of Extension Studies, University of Papua New Guinea, Waigani, 1980.

11. Kaeley, G.S., "Teaching Mathematics at First Year Level to Mature Age Students, at the University of Papua New Guinea", (mimeographed), Department of Extension Studies, University of Papua New Guinea, Waigani, 1982.

12. Keegan Desmond J., "From New Delhi to Vancouver : Trends in Distance Education" in Daniel J.S. *et al* (eds), *Learning at a Distance - A World Perspective*, Athabasca University/ICCE, Education, 1982.

13. Keegan Desmond J., *Six Distance Education Theorists*, ZIFF, Hagen, 1983.

14. Kember David, "Some Factors affecting Attrition and Performance in a Distance Education Course at the University of Papua New Guinea" in *Distance Education*, Vol.2, No.2, Open University, Milton Keynes, 1980.

15. Kerlinger Fred N., *Foundations of Behavioural Research*, Holt, Rinehart and Winston, Inc., New York, 1967.

16. Kidd J.R. in the Foreword to Ignacy Waniewicz, *Demand for Part-time Learning*, Educational Communication Authority, Ontario, 1976.

17. Leedy Paul D, *Practical Research Planning and Design*, Second Edition, Macmillan Publishing Co. Inc., New York, 1980.

18. Mackenzie Norman, *et al*, *Open Learning*, the Unesco Press, Paris, 1975.

19. Mainusch Herbert, "How can the Conventional University Serve the Distance Learners", in Daniel J.S *et al*, (eds), *Learning at a Distance - A World Perspective*, Athabasca University ICCE, Edmonton, 1982.

20. Mcintosh V. *et al*, "Student Demand, Progress and Withdrawal - an analysis of the Open University's first four years" in *Higher Education Review*, Autumn, 1974.

21. Mmari C.V., "The Implementation of the Musoma Resolution. The University of Dar es Salaam Admission Experience" in Ishumi A.G.M. *et al*, *Education and Social Change*, Black Star Agencies, Dares Salaam, 1980.

22. Mauly George J., *The Science of Educational Research*, American Book Company, New York, 1963.

23. Mowbray David, *et al*, "Performance of PY students compared to Grade Ten Results", (mimeographed), UPNG, Waigani, 1981.

24. Murphy P.S., "National High School versus Preliminary Year Studies: Performance of UPNG Foundation Year Students 1979" in *Papua New Guinea Journal of Education*, Vol.17, No 1, Port Moresby, 1983.

25. Murphy P.S., "UPNG as an agent of social stratification : an analysis of output by sex". A paper presented at the Extraordinary Meeting of the Faculty of Education, Waigani,

26. Nelson A.J.A., "The Universities and Adult Education" in Whitelock Derek (ed), *Adult Education in Australia*, Pergamon Press, Australia (Rushcutters Bay, NSW 2011), 1970.

27. Perraton H., "Distance Teaching North and South" in Daniel J.S. *et al* (eds), *Learning at a Distance - A Word Perspective*, Athabasca University/ICCE, Edmonton, 1982.

28. Ros Lawrence, "Transition Rates and Mathematics Achievement in Five Developing Countries", Goroka Teachers College, Goroka, (mimeographed), undated.

29. Sewart David, "Individualizing Support Services" in Daniel J.S. *et al* (eds), *Learning at a Distance - A World Perspective*, Athabasca University/ICCE, Edmonton, 1982.

30. Taylor J.C. *et al*, "Student achievement as a function of perceived study time : a comparative analysis" in Daniel J.S. *et al* (eds), *Learning at a Distance : A Word Perspective*, Athabasca/ICCE, Edmonton, 1982.

31. Tight Malcolm, *Part-time Degree Level Study in the United Kingdom*, The Advisory Council for Adult and Continuing Education, Leicester, 1982.

32. West Leo H.T., *et al*, "Reasons for Entry into Higher Education" in Hore T. *et al* *Mature Age Students in Australian Higher Education*, Higher Education Advisory & Research Unit, Monash University, 1980.

33. Whitelock Derek, "*A Brief History of Adult Education in Australia and an outline of current provision*" in Whitelock Derek (ed), *Adult Education in Australia*, Pergamon Press, Australia (Rushcutters Bay, NSW 2011), 1970.

34. Wilson Lascelles, "Australian Adult Education compared with Overseas Systems" in Whitelock Derek (ed), *Adult Education in Australia*, Pergamon Press, Australia (Rushcutters Bay, NSW 2011), 1970.

35. Zahn J.C., "Differences between Adults and Youth affecting learning" in Kernes F.A. *et al* (eds), *Issues and Trends in Adult Basic Education, Focus on Reading*, University Press of Mississippi, Jackson, 1980.

5. MATHEMATICS TEACHING IN VARIOUS CULTURES

WHAT IS IT LIKE BEING A MATHEMATICS TEACHER IN A SECONDARY SCHOOL IN PAPUA NEW GUINEA

Naomi Wilkins

Mathematics Education Centre

ABSTRACT

Some of the many problems that a secondary mathematics teacher in Papua New Guinea faces are shocking as well as frightening.

The teacher's guides and syllabi are usually very brief with few examples for teacher to see. There is a lack of materials and teaching aids. Teachers are overloaded with teaching and extra curricular duties. This results in not enough time allocated towards proper lesson planning, and preparation of materials and teaching aids. Forty three students in a class is too many for a teacher to handle. It does not allow enough time to be given to each individual student. The students feel left out many times.

If a room or two are allocated towards mathematics subject to be taught in it would be good. Materials and aids could be kept in one central place for use by all grades.

Not only teacher face problems, but students as well. They have problems at home with their parents. Some have health problems. Students have daily timetables or schedules that they follow which are usually very tight. This tires them out so they can not concentrate in the classrooms.

Many students struggle hard trying to understand English and mathematical words. Others are confused try to relate their way of working out mathematics to the school's way of working them out.

INTRODUCTION

I taught for only two years and one term at Bugandi Provincial High School so I am not talking as an experienced teacher. However I believe that most of what I say is true for most national secondary mathematics teachers in secondary schools in Papua New Guinea.

THE TEACHER'S ROLE IN PROMOTING THE PUPILS' GROWTH IN MATHEMATICS

One important aspect of being a teacher is to recognise and deal with each pupil according to his or her needs. This was not easy especially when there was an average of forty three students in a class. I used to take about 42 pupils in each class. I used to find it very difficult in one single lesson to help every student, even if I put them into groups.

Usually I would set them exercises to do. While they were doing them I would go around and help those who needed help. I used to encourage the students to come and see me after school if they had any problems about what I was teaching them. Only a few who cared or wanted help would come and see me. Most of the time I had to go over what I had taught in the last few lessons.

I found that there were students who could not participate easily because they were either having difficulties with the work and felt they were being left out, or because they did not know how to speak up. Others were embarrassed to speak and express their opinions. Many students do not express themselves openly in class. This occurs because in Papua New Guinea some societies have a custom where the youngsters are taught to keep quiet and not talk to the elders. It would be disrepectfull.

This kind of behaviour is very hard to change overnight. I often had to talk them into contributing ideas in class.

Another thing a teacher does is develop students' attitudes and skills necessary for effective participation. The students participated more in varied practical activities (whether individually or in groups) than when theory work was being taught. Exercise after exercise bored them because they could not move around as they did during practical activities.

Helping the children aquire the values realised as ideals of democracy is what the teacher has to do: values such as mutual respect, willingness and ability to co-operate in the solution of problems.

I found that the children I taught on the whole liked me and respected me. Most children I taught mathematics liked the subject and were willing to learn. Often the children would want to solve the mathematics problems either with their close friends or with friends they knew in the classroom. I never stopped anybody that left his desk to go and seek his friend's help from the other end of the classroom as long as they were quiet.

I felt this was the secret, leaving the students to solve their own problems instead of spoon feeding them all the time. I felt that if I wanted the children to learn more I had to break loose their power of hiding away in their own little world and coming out and participating in discussion and correction of work.

I used to take some of my mathematics class children for expressive arts, dance and drama as well. I found that those students were more open and more expressive during mathematics classes. That made it easier to teach them. They were willing to learn. Perhaps getting to know them in the freer atmosphere of a dance class helped the mutual relationship.

SELECTION AND USE OF A WIDE VARIETY OF INSTRUCTIONAL MATERIALS

One is told at inservice meetings, departmental meetings and in the Educational Gazette over and over again that effective teaching occurs when teachers select and use a wide variety of instructional materials. But are those materials available? Many of the materials mentioned in the teachers guides and the students units book were in fact not available. I used to try and work out something similar for the students to use. However at times I had to just mention the materials and explain briefly what we would have done if the materials had been available. I would just give them exercises to do then.

If there were any materials available at all, many times there were not enough of them to use in a class.

I often wondered how much the children missed out on because of the scarcity of materials. I was teaching at one of the biggest schools in Lae. One would imagine that a rural school would be worse still.

Because of the lack of teaching and learning aids, most of my teaching was theory work. I believed that more practical work in teaching would have promoted more learning and understanding of mathematics.

There was popular saying in the schools "Make use of just what is available". In most cases however there was nothing available.

Teacher's guides and syllabi were usually very brief. Often they were not clear. Sometimes they were too difficult to interpret and plan lessons properly. If examples of exercises for each topic were included, they would have been very helpful.

PREPARATION TIME AVAILABLE

Providing effective and continuing motivation is another aspect of a teacher's job. Very often I felt that more effective teaching would have occured if teachers were allowed enough time to prepare materials and aids. At the school I was teaching at, we would normally have to be at the grounds at seven o'clock for the morning assembly. We would then start classes at seven forty. The classes would then end at one . thirty in the afternoon with half an hour break in the morning at ten twenty.

After lunch at two thirty we had to be back at the school grounds for extra curricular activities. They were either sports or work parade supervision. Those would end at four fifteen. Some teachers who were expected to supervise the study periods in the evenings as well.

A junior teacher would have about thirty three x 40 minutes periods of teaching load every week from a total of 39 periods. The remaining six periods were devoted to administration tasks as well as preparation and marking. I did 33 periods in the first 2 years and 32 in the third year. There were not many free evenings for a dedicated teacher.

STUDENTS' DAILY ACTIVITIES

The students used to get up before six in the morning to clean their dormitories and have their shower. They would then go and have their breakfast. The bell would be rung at seven for the morning assembly. The school would start at seven forty with the break at ten twenty which lasted for half an hour. The classes would end at one thirty in the afternoon for lunch. The bell would go at two thirty in the afternoon for the assembly. There the students would either be assigned to go and do various activities or play sports. The bell would go again at four fifteen for the end of work parade and/or sports. At this time they would go and get ready for tea at five o'clock. After tea at seven o'clock they would go for night studies. The bell would go at eight thirty for the end of studies. At nine o'clock they would go to bed at which time the lights would go out. In all the school day was quite full for the students.

SOME PROBLEMS FACED BY STUDENTS

The students at the school had come from all over the country, with the majority from the Morobe Province. Their level of knowledge differed considerably. If a student came from one of the town or main outstation schools he would most likely be a better student than one that came from a rural school. I had to look at this aspect very closely when I was teaching. Their way of thinking mathematically was very much dependent on their background, language, cultures and the kinds of environment they had grown up in. There weren't any support from parents with their maths problems. The children had to depend on us alone.

Some children did not progress as fast as others because of their state of health. For instance, some students had sores, scabies and/or lice in their hair. Others were sick most of the time from bacterial infections. Personal was always being emphasized in the school, but some boys took a long time to respond.

The grade seven students found it very difficult to adjust to the school's hectic life. Many who came from the rural schools found it impossible to concentrate often, because they used to miss their parents. This was a simple case of homesickness, but some students took a long while to overcome it.

Many students felt hungry in periods seven and eight because lunch hour began at one thirty instead of twelve o'clock, the time they were used to. This affected their ability to concentrate.

A number of students were worried because their parents could not afford to pay off their school fees. This would mean they would be terminated because the school would not keep them.

Those who were day scholars often went hungry because their guardians or parents did not give them breakfast before they came to school. Some had unsettled parents and that affected their classroom learning.

MATHEMATICS AND LANGUAGE PROBLEMS

Many English words and mathematical terms were often the cause of the difficulty in the learning of mathematics. Many of the English words used were often entirely new to the children. Children who were not keen in learning lost interest quickly because they did not understand the words I was saying. Fortunately there were few students who were not keen.

The environments the students grew up in heavily influenced the way they thought and did things mathematically. Their methods of solving certain problems were totally different from the methods used in schools. The children were confused many times. For example in Papua New Guinean to talk of sharing an orange between Peter, Paul and Mary would be something like this: half for Peter, half for Paul and half for Mary, the 'half' really meaning 'any size. In school the above case would mean one third for Peter, one third for Paul and one third for Mary. In English for measurement of heights and distances we use: tall for heights and long for distances. In Papua New Guinea the word long would be used for the both measurements. Hence confusion for the students.

In many cases mathematical topics that were taught in schools were entirely new to the students. They rarely had been in situations which mirrow mathematical situations outside of the classroom. Hence they had little to refer their new knowledge of mathematics to.

SEX

The school I taught in was a male students only school. One advantage of a non co-educational school is that the students in all grades participate easily in class as well as outside of class. There are none of the opposite sex to worry about or get embarrassed about, at least not in school. This does not only help in mathematics, but in other subject areas as well.

The disadvantage of this situation is if you are young and single, whether you are a male or a female. You have to be very careful when playing your role as a teacher. I taught only the junior classes in my first year out teaching. It was in my second year of teaching that I actually taught a senior class. I found that it was easier to work with the junior students than the senior students. The main reason for that was many of the senior students I was teaching were almost as old as me, and that caused them to get embarrassed often and not participate easily. This is probably far more of a problem in Papua New Guinea because of the very behaviour patterns expected by the traditions of the country. Men are simply not taught by women.

MATHEMATICS ROOMS ALLOCATIONS

At the school there were not rooms allocated to major subjects like Mathematics, Science, Social Science and English. The students remained in the classrooms while the teachers moved around between periods. Most of the classrooms were nearly bare. At the most a few posters, charts and some of the students' work would be pinned up. Many of the aids that were used were taken back to the individual teacher's office and left there until needed. If the teachers remained in the classrooms while the students moved around the teachers would have cared more about putting displays of aids, posters, pictures, students best work, magazines and so on. This would have helped the students learn more. I think it would have been a good thing if there had have been an allocation of one or two mathematics classrooms where all the mathematics lessons would take place. This way all the aids and materials would be in one place for all the students to use.

CONCLUSIONS

The average number of students to a class was 43. This was too big for one teacher to handle in one single (40 minutes) periods, helping each individual student with his/her problems.

Materials were rarely available for use in classrooms. Teaching and learning aids were also not available. There was too much theory rather than practical work completed due to the scarcity of the materials.

Teachers were often overloaded with extra curricular duties. This results in poor teaching performances through lack of proper planning and preparations.

Students background, cultures, etcetera differs alot. These should be taken into consideration when teaching them. It was noted that students that came from the rural schools did poorly when compared with those who came from main outstations or town schools.

Often English words used in mathematics are difficult. for the students to understand. Many mathematical terms are difficult too. Students struggle just to remember the terms, rules and formuli in mathematics. As a result the students can not apply them easily to other similar mathematical problems.

Pidgin and Hiri Motu have a limited number of words. A word in these languages can stand for more than one western concept. English in comparison is a much more specific language.

Both the syllabi and the teacher's guides are usually very brief. Many times teachers are not clear of what to teach. Examples of exercise on each topic should be included.

Despite the problems of teaching mathematics in a Provincial High School, I did enjoy it. The interaction with the students and the opportunity of helping them develop as mature individuals was always a stimulus.

I was fortunate in having two very helpful Heads of Department in the time I was teacning. The second of the two went so far as to come and observe my lessons. During this time he would make helpful criticisms and give enouragement. This was useful because it gave me a lead in the way I could improve my teaching. He did this with all the teachers in the Department. During the weekly hour-long Department meeting he always held more general comments about our teaching were made.

Responsibilities within the Department were shared among the staff. Therefore we all learnt how to standardise students' mark and rank them. This was done at the end of each term. Thus a good ongoing record of students' achievements was kept. Taking such responsibilities was good practice in case we ever become Heads of a Department. It also meant we were involved with the administration of the school and not just teachers.

The combination then of knowing you were doing a worthwhile job, and working with a supportive and cooperative staff enabled me to overcome alot of the frustrations I have talked about before.

MATHEMATICAL ABILITIES AT THE UPPER SECONDARY SCHOOL IN FIJI

I L RAJ

U.S.P.

ABSTRACT

The paper reports on research being currently conducted in 10 secondary schools in Fiji, and is concerned with the investigation of mathematical abilities at the upper secondary school. A brief outline of the design of the research with respect to the measurement of mathematical ability is presented.

INTRODUCTION

Since 1971, a locally produced curriculum in secondary school mathematics has been implemented nationally in Fiji. All textbooks are also produced locally. This however only goes up to Form IV level. In Forms V and VI, students work with overseas curricula and are prepared for overseas examinations. (See Appendix).

Students moving beyond Form IV, then, find themselves shifting from a local-based setting to a possibly more academic and pure curriculum which may leave many students feeling less secure in a wider field.

This happens at a time when many students are beginning to form more serious attitudes towards their studies and making up their minds as to where their interests really lie.

Teachers, too, start working under pressure at these levels in order to complete the syllabus in the limited time available and are vitually forced to adopt 'spoon-feeding' techniques to cram as much as necessary in. Students are taught rigid methods, almost by rote, to solve 'nicely constructed' problems and very little time, if any, is afforded to the logical construction of the components required in solving a problem. As a results, students begin to show reluctance to do much unless they are given precise step-by-step instructions and very few are willing to make the effort to think their way through a problem. Often, interest is lost and enthusiasm with the subject fades.

In order to look more closely at this situation, this research is concerned with an investigation into the mathematical abilities and attitudes towards mathematics at the upper secondary school level in Fiji.

This paper outlines briefly the design of the research with respect to the measurement of mathematical ability, being carried out at 10 secondary schools in Fiji.

Design

In developing the test a taxonomy model was used, based on the Begle Wilson taxonomy used in the NSLMA-Project (1, p 14) consisting of a 12 cell matrix with four different cognitive levels and three general areas of mathematical content. This is shown in Figure I.

MATHEMATICAL CONTENT

Cognitive Levels	Number Systems	Geometry	Algebra
I. Computation			
II. Comprehension			
III. Application			
IV. Analysis			

A brief descriptions of the content and cognitive levels associated with the matrix are given below. These are the levels described by Begle:

Cognitive Behavioural levels

Computation - Items designed to require straightforward manipulation of problem elements according to rules the subjects presumably have learned. Emphasis is upon performing operations and not upon deciding which operations are appropriate.

Comprehension - Items designed to require either recall of concepts and generalizations or transformation of problem elements from one mode to another. Emphasis is upon demonstrating understanding of concepts and their relationship and not upon using concepts to produce a solution.

Application - Items designed to require (1) recall of relevant knowledge, (2) selection of appropriate operations, and (3) performance of the operations. Items are of a routine nature. They require the subject to use concepts in a specific context and in a way he has presumably practiced.

Analysis - Items designed to require a nonroutine application of concepts.

Mathematical Content

Number Systems - Items concerned with the nature and properties of whole numbers, integers, rational numbers, real numbers, and complex numbers; the techniques and properties of arithmic operations.

Geometry - Items concerned with linear and angular measurement, area, and volume; points, lines and planes; polygons and circles; solids; congruence and similarity; constructions; graphs and coordinate geometry; formal proofs; spatial visualization.

Algebra	-	Items concerned with open sentences; algebraic expressions; factoring; solution of equation and inequalities; systems of equations; algebraic and transcental functions; graphing of functions and solution sets; theory of equations; trigonometry.

The cognitive levels are arranged in heirarchical fashion. This is described by Wilson (2, p 649):

> "The four cognitive levels of behaviour are both heirarchical and ordered. They are ordered in the sense that analysis is more cognitively complex then application, which in turn is more cognitively complex than comprehension, and the computation level includes those items which are the least complex.
>
> It is heirarchical in that, for example, an item at the application level may require both comprehension level skills and computation level skills".

The 'mathematical ability' being considered was categorized into:

(i)	School Ability	- the ability to handle the mathematics which is taught at school. Learning and mastering the skills and rules and knowing how to use them in routine problems associated with the syllabus.
(ii)	Creative Ability	- the ability to analyse and apply learned skills and rules to non-routine problems, to reason creatively and show inventiveness.

Both psychologists and mathematicians normally distinguish these two levels. Krutetskii (4, p 67) uses the same terms. R. M. Gagne and N. E Paradise (4, p 68) refer to 'reproductive' and 'productive' mathematical activity. The terms 'lower mental processes' and 'upper mental processes' are also used.

School ability in this research is regarded as consisting essentially of cognitive levels I and II with an overlap into III, while creative ability consists mainly of cognitive levels III and IV with aspects of II included.

Specific Aspects

Two problem sheets were constructed. One at Form 4 level and one at Form 6 level. Each comprised 24 items. Two items were designed for each cell of the matrix each of which was homogeneous with respect to both content and cognitive behaviour required to work the item at the respective level.

The items consisted of

(a) Multiple Choice Questions - for short tasks

(b) School type problems - longer tasks asking for any necessary working

(c) Creative, non-routine type problems, reading proofs - all working or explanations asked for.

The time given for the set of problems was 1 hour.

Test Marking

Each of the 24 items were equally weighted with a maximum score of 3 points.

The multiple choice questions contained one or two options which were reasonably close to the correct answer. A student selecting one of these was given a score of 1. The correct answer scored 3 and anything else 0.

In school-type problems, the answers were marked in the usual school manner with a student scoring 0, 1, 2 or 3 depending on how close to the final solution he/she got using the learned techniques and algorithms.

A common criticism of taxonomies is that a person who gives an incorrect answer to a problem at a high level may have mastered the intended difficulty but has failed, e.g. due to a computational mistake.

In order to overcome this problem, performance in the analysis problems was measured in terms of interaction with the student and the particular problem, using a scoring scale based on the Rasch system (3, p 206). Here each student's response is scored according to how close it is to a valid solution using the following 4-point scale.

Score	Solution Stage
0	Noncommencement The student is unable to begin the problem or hands in work that is meaningless.
1	Approach The student approaches the problem with meaningful work, demonstrating some understanding of the problem, but an early impasse is reached.
2	Substance Sufficient detail demonstrates that the student has proceeded toward a rational solution, but major errors or misinterpretation obstruct the correct solution process.
3	Result and Completion The problem is solved or very nearly solved. There may be a few minor errors but the student has clearly understood the way to solve the problem.

With this scoring system it was possible for a student to score full marks on an item in the analysis level even though minor errors produced an invalid solution.

A student's overall score on the problem sheet was taken to be a general indicator of his/her ability in mathematics as defined above.

Inferences about peformance of the different cognitive levels and in the areas of context will be investgated and will be interpreted in conjunction with the results from the attitude measures.

APPENDIX

Summary of External Examinations taken by Secondary School students in Fiji (5, p 105).

a) *Secondary School's Entrance Examination*

This is taken in Class 8 i.e. in the eight and last year of primary school. Admission to Form III in Government and grant-aided secondary schools is based on the results of this examination. The paper is set locally.

b) *Fiji Junior Certificate Examination*

This is normally taken in Form IV, i.e. at the end of the second year of secondary school. It is based on Form III and Form IV work. The paper is set locally.

c) *New Zealand School Certificate Examination*

This is taken in Form V. The paper is set overseas but a special South Pacific option is taken.

d) *New Zealand University Entrance Examination*

This is taken Form VI. Paper is set overseas.

e) *Fiji Seventh Form Examination*

The syllabus is based on the New Zealand 7th Form Pure Maths Bursaries syllabus, and is set locally.

REFERENCES

1. BEGLE, E. G. Critical Variables in Mathematics Education. Findings from a survey of the Empirical Literature Mathematical Association of America and the National Council of Teachers of Mathematics, Washington, D.C. 1979

2. BLOOM, B.S., HASTINGS, J.T. and MADAUS, G.F Handbook on Formative and Summative Evaluation on Student Learning. New York, McGraw-Hill, 1971

3. KRULIK, S. (Editor) Problem Solving in School Mathematics National Council of Teachers of Mathematics, Reston, 1980

4. KRUTETSKII, V. A. The Psychology of Mathematical Abilities in School Children. The University of Chicago Press, Chicago and London, 1976

5. SHERLOCK, P. et al. Education for Modern Fiji. Report of the 1969 Fiji Education Commission Department of Education, Suva, 1969.

The Effects of Consumer Mathematics
on Basic Skills Computations
at Yap High School

Dale Nelson
Mathematics Curriculum Office
Department of Education, Yap
Federated States of Micronesia

The Effects of Consumer Mathematics on Basic Skills Computation at Yap High School

The education system in Yap is modelled after the United States' system causing Yap High School to carry grades nine through twelve. Before two years ago the standard mathematics courses in ninth and tenth grades were adapted from an eighth and ninth grade general mathematics curriculum in the States. The students' performance was retarded yet another year or so. Consequently, a majority of the students graduated without the ability to perform basic operations involving mathematics.

To remedy this deficiency, the Department of Education undertook a two-part program consistent with the "Agenda for Action" advocated by the National Council of Teachers of Mathematics. The first part aimed at improving the teaching of problem solving throughout the curriculum. The second part introduced a consumer class at the high school that directly attacted problems confronting our unprepared graduates. This paper is concerned with the effects of the second part of that program.

The ninth and tenth grade general mathematics classes were combined into a single general class for the ninth grade. A new ninth grade curriculum was adopted from the States to replace the lower curriculum that had been in use. For the tenth grade, the Mathematics Department suggested writing a consumer mathematics program which emphasized Yapese life style. We suggested eliminating the computation exercises that had been in the general curriculum and using only word problems for the exercises. We felt that an intensive study would give students the best chance to learn consumer application problems. We also thought that this approach might provide incentive to master computational skills as they would be embedded in realistic and familiar problems.

Although it was agreed that a consumer mathematics class was needed, the approach we suggested seemed unsatisfactory to many people. Since Yapese read English as a second language, they feared that the extensive use of word problems would be a barrier to the students' understanding. Also, the students were not at a high school level; they were deficient in general math skills. People wondered if their inadequate skill might even deteriorate when the computation exercises were eliminated and the students stuggle through word problems to do the computations.

However, it was finally agreed that the intensive program would be initiated. It was decided that a series of progress tests used in the tenth grade general mathematics classes would be continued. If the students' computation skills degenerated in the new class, adjustments would be made.

The tests had been given to measure students' progress in basic computational skills through each academic year. They were essentially sixth grade arthmetic tests containing a set of twenty problems. There was at least one problem for each operation in each of the non-negative number systems: whole numbers, decimals and fractions. The tests were given during

BEST COPY AVAILABLE

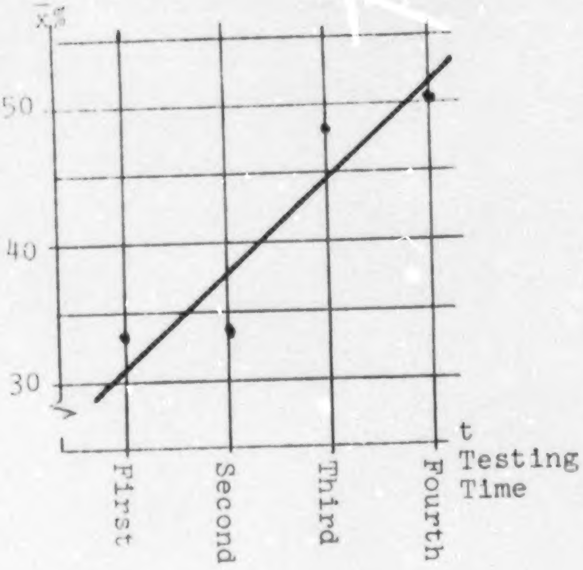
forty-five minute class periods, and the scores were reported as percents.

The following table shows the results of the progress tests given the last two years of the general mathematics class in the tenth grade. $\bar{x}$ is the mean percent scored each testing time, n is the number of students who took the test and s is the standard deviation.

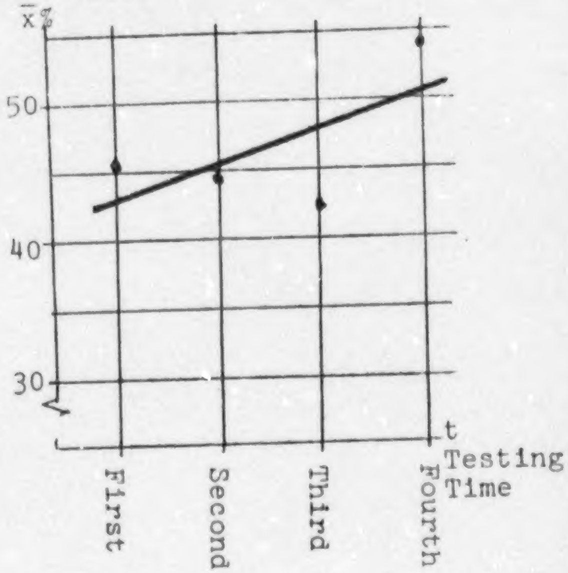
Testing Time year	First	Second	Third	Fourth
80-81	$\bar{x} = 33.38$ $n = 16$ $s = 15.8$	$\bar{x} = 34.05$ $n = 21$ $s = 19.4$	$\bar{x} = 48.42$ $n = 19$ $s = 21.8$	$\bar{x} = 50.42$ $n = 19$ $s = 21.1$
81-82	$\bar{x} = 45.27$ $n = 77$ $s = 24.0$	$\bar{x} = 44.69$ $n = 78$ $s = 24.1$	$\bar{x} = 42.94$ $n = 77$ $s = 20.6$	$\bar{x} = 54.43$ $n = 77$ $s = 17.7$

These averages are quite low for high school students on a test of the type. We had expected the average at the beginning of each year to be close to 50%. By the fourth test we anticipated the average would be between 75% and 80%. One reason for the low scores is that Yap High School is open to practically everyone who has passed the eighth grade, and in Yap that is determined more by attendance than by scholarship. We would like to change this, but it is the situation that we deal with at present. The size of the standard deviations indicates a large range of scores. Many of the scores are good, but some are less than 20%, and a few are zero.

These averages are shown in the following graphs. The regression lines are also shown to indicate the rate of progress.



80-81



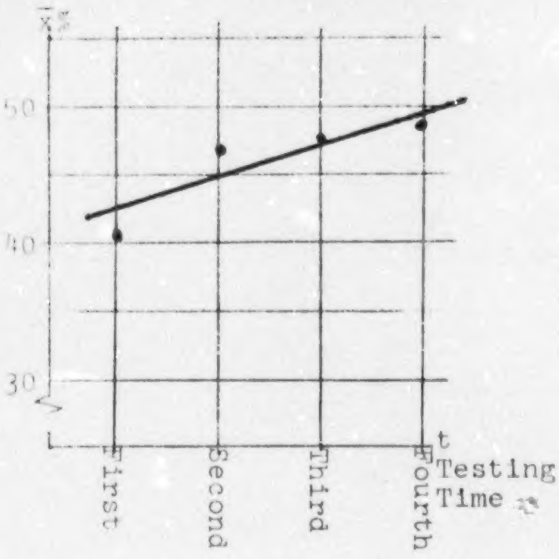
81-82

$\bar{x} = (6.55)t + 25.2$ is the equation for the 80-81 regression line; the correlation coefficient is 0.93. The regression coefficient 6.55 is an accurate indication of the rate of improvement.

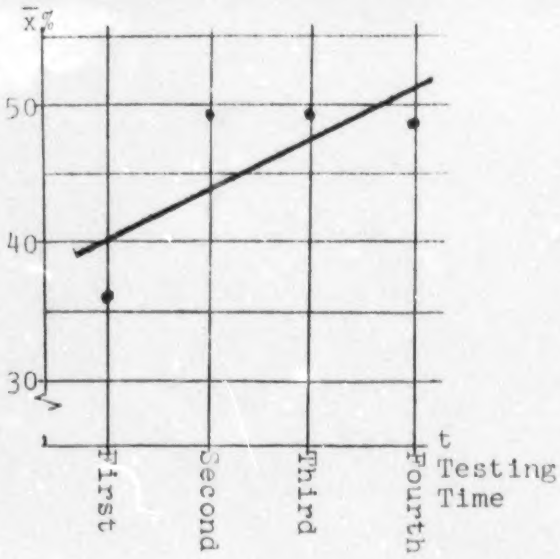
$\bar{x} = (2.57)t + 40.4$ is the equation for the 81-82 regression line; the correlation coefficient is 0.64. The mean for the fourth tests for these two years is $\bar{x} = 53.64$; $n = 96$ and $s = 18.4$.

The second table shows the results of the progress tests given the first two years of the consumer mathematics class in the tenth grade.

Testing Time year	First	Second	Third	Fourth
82-83	$\bar{x} = 40.68$ $n = 74$ $s = 19.0$	$\bar{x} = 46.94$ $n = 72$ $s = 19.0$	$\bar{x} = 47.46$ $n = 60$ $s = 20.2$	$\bar{x} = 48.52$ $n = 77$ $s = 19.3$
83-84	$\bar{x} = 35.75$ $n = 30$ $s = 23.4$	$\bar{x} = 49.13$ $n = 32$ $s = 26.7$	$\bar{x} = 49.19$ $n = 31$ $s = 24.8$	$\bar{x} = 48.79$ $n = 29$ $s = 24.2$



82-83



83-84

$\overline{x} = (2.40)t + 39.9$ is the equation for the 82-83 regression line. The correlation coefficient is 0.88.

$\overline{x} = (3.92)t + 35.9$ is the equation for the 83-84 regression line. The correlation coefficient is 0.76.

The mean of the fourth tests for these two years is $\overline{x} = 48.59$; n = 106 and s = 20.63.

Comparing the mean from the fourth tests in the general mathematics classes with those from the consumer classes, a t-value = 1.83 is obtained. Within an error propability of 5%, the two means differ only randomly.* The replacement of computation exercises with applied word problems has had no significant effect on the students' ability to work basic computations. There is no reason to make adjustments is the present consumer mathematics curriculum to compensate for the eliminated computation exercises.

We are confident that the new program is not worse than the old curriculum that it replaced, as some people had feared. However, our data does not show that the students are better at consumer problems. When we realized that consumer application hadn't been adequately taught, we intensified its teaching without first testing the degree of ignorance. Had we tested the general classes on consumer applications, we believe that they would have shown overwhelming ignorance. We also suspect that the students have better retention of basic computation skills because they use them in realistic and familiar problems. But when we began our testing, we hadn't planned to make so many conclusions. The only conclusion that we can justify is that under the new program, students have retained skill in basic computations

---

* t-value $\geq$ 1.98 would indicate that the means differ significantly with an error probility of 5%.

BEST COPY AVAILABLE

AN INNOVATION IN SECONDARY MATHEMATICS TEACHER EDUCATION IN UGANDA

M.M. MAGINO
MINISTRY OF EDUCATION
UGANDA

Mr. Chairman invited guests and fellow Mathematics Educators. It is a great honour for me to have this opportunity to present, a short paper on Secondary Mathematics Teacher Education in Uganda, at this conference of international importance. A conference of this nature is regarded as a landmark in the field of Mathematics Education development; it is also a landmark in the field of curriculum development which begins with identification of a problem, analysing it and soliciting news from a cross section of the society. A meaningful and relevant curriculum is that constructed on the basis of the concerted efforts of the practising teachers, teacher trainers, University lecturers, education administrators, informed members of the public and curriculum developers.

Mr. Chairman, allow me to convey greetings from the people of Uganda to the people of Papua New Guinea. In particular I bring greetings from the Honourable Minister of Education, the P.S. Ministry of Education and my Chief Inspector of Schools who I work under.

Mr. Chairman Mathematics Education in Uganda has suffered several setbacks over the decade. In 1972 we experienced a massive exodus of expatriate teachers who were manning the the whole curriculum and this left only a handful of qualified teachers in Secondary and Primary Schools. The government had to work fast. The intake at the National Teachers College, Kyambogo was stepped up, to alleviate the shortage. However this has not been very satisfactory.

Mr. Chairman in January, 1978 a Mathematics conference was convened in which participants were invited from a cross section of mathematics educators. The main reason that led to the convening of this conference was the outcry of the public on the state of Mathematics Education. The poor performance in final examinations, the lack of sufficient, elaborate teaching materials, the lack of detailed teaching syllabuses were cited as a few of the reasons that bought about the dissatisfaction among the public and the professionals alike.

The Minister of Education, then, got deeply concerned with the improvement of Mathematics Education and therefore called upon all mathematics educators and Mathematical Association to examine critically the type of Mathematics taught in schools at that time and recommend ways and means of improving the subject.

He said:-

In considering the type of Mathematics that must be taught in Uganda Schools you should bear in mind the following goals of teaching Mathematics.

1. Mathematics is a utilitarian subject. It is useful in everyday life in counting our property such as cows and money in measuring and weighing different things, in buying and selling our produce etc.

2. Mathematics is useful in solving social problems. For any problem in real life situation Mathematics provides a solution by translating it into a model, setting up an equation using mathematical techniques.

3. The cultural component in Mathematics depicted by the historical development of the subject in varying cultures must be maintained.

4. Mathematics must provide for personal interest of the learner of the subject.

Among the many recommendations made at the close of the conference, training and retraining of teachers was the first and most important step the government should look into. The conference said the following.

It is noted that there are not sufficient trained and qualified teachers to effectively teach the subject at all levels in all schools. This contributes a great deal to the poor teaching and performance in examinations. Hence a crash programme should be carried out to retrain teachers, and train licenced teachers throughout the country.

It is also noted that there were not enough teachers to teach the double mathematics at A Level. It is therefore recommended that the Faculty of Education in collaboration with the Mathematics Department at Makerere University and the National Teachers College should be called upon to train these teachers in sufficient numbers.

The conference went on to further note that the National Teachers College had been and still is the main source of secondary school teachers. In view of this and in order to increase the number of qualified and trained teachers, the group strongly recommended that at N.T.C. Mathematics should be offered as a "Double main subject". It was realised that the "Double main" would help to alleviate the shortage of teachers in other subjects. By making Mathematics a Double main subject students will have ample time to concentrate and specialize in Mathematics.

At that conference it was also realised that there was a great need for more graduate teachers of Mathematics. It was noted that the Diploma holders had been performing well in their degree courses. Also the programme at N.T.C. covers a lot of the topics in the degree programme. It was the opinion of the participants that the students who perform well at N.T.C. should be given direct entry into the University to get higher qualifications in Mathematics in order to become better teachers.

MATHEMATICS CURRICULUM

Much curriculum development has taken place in many subjects in Uganda Secondary Schools. Mathematics syllabuses have also changed since 1961. It is always the hope of the architects of such changes that they will be for the better. In our schools there are up to date, four mathematics syllabuses: syllabus A, B and S and the new syllabus coded 456 by the Uganda National Examinations Board (UNEB). This last one is a cocktail of syllabuses B and S. It was examined for the first time in 1983. The A,B and S have now been phased out. This year 1984 and on there will be only one syllabus for the Uganda Certificate of Education.

All these changes are called for to improve the performance of candidates not only in public examinations and identification with syllabuses used in the other Countries but also in the field of employment. The standard of performance in Mathematics in Secondary Schools is on the decline as observed from the school certificate examination results. As already mentioned, parents,learners and other interested parties have voiced dissatisfaction on the performance.

The National Curriculum Development Centre (NCDC), in conjuction with the Central Inspectorate organised the conference I have already talked about.

The deliberation at this conference were not followed up until after the elections of 1980 in which the present government came into power. The government went ahead to expand the training of teachers. As a result new ways of training the teachers in Mathematics have been developed.

In 1982/83 academic year a college which is an extension of National Teachers College was started at Ngetta Lira in Northern Uganda. This College was started with the sole purpose of training Grade five teachers taking a "Double main" course in mathematics. A syllabus: "Mathematics Double Main" was worked out under my chairmanship to cater for this course. The University Senate examined it and approved it. This syllabus attempts to solve the problem of producing competent personnel.

At the moment the teachers supplied through the single main are divided into two i.e. they serve two masters and therefore may not teach the subject as effectively as required.

It is therefore believed that on the introduction of a "Double Main" course teachers will be very competent and confident to teach the subject very effectively.
It must be noted that the University produces teachers of Mathematics but these cannot satisfy the demand in schools. It is hoped that with the introduction of this syllabus at N.T.C. Ngetta Lira the teachers so produced will be better than those who have taken a single main. They are also expected to alleviate the problem at advanced level (HSC).

The final syllabus is a result of consultations with famous mathematics educators, curriculum experts, practicing teachers and lecturers of mathematics and educationists at the University.

It is also our hope that the syllabus is an antidote to the lack of competent mathematics in schools and hence poor performance in public examinations.

SECONDARY TEACHER EDUCATION

By 1981 there were only two institutions in Uganda that prepared secondary school teachers. The National Teachers College Kyambogo started in 1965 as a separate college solely to train non graduate teachers, now offers a two year programme for students who have completed S6 and a three year programme who have completed S.4,though this latter option was phased out in 1975.

Makerere University trains secondary school teachers at both the undergraduate and postgraduate levels. The under graduate programme is taken concurrently with students normal degree programme while the postgraduate programme is a years course. Both of these programmes award the Makerere Diploma in Education.

The output from these two institutions has dramatically increased since 1980 so that today Uganda's secondary schools are fully staffed with qualified teachers in the Arts subjects. There is still a shortage of teachers in Mathematics and Physics, but these shortages should be overcome in the next few years given that the secondary schools and training institutions have been expanded. The government has opened up three other National Teachers Colleges at Ngetta in Lira, Kakoba in Mbarara and Nagongera in Tororo beside the one at Kyambogo. N.T.C. Ngetta is specially for Mathematics while Kyambogo and Kakoba offer Math as a single main.

SYLLABUS FOR MATHEMATICS DOUBLE MAIN COURSE AT NGETTA

This was a most welcome development which if followed up should reduce our shortage of mathematics teachers in the post primary institutions. The coverage of the syllabus is ambitions and straight away demands enthusiastic graduate mathematicians with a liking for teaching the subject.

We need a course that will introduce students to machines and their operations: a watch, a motor, a telephone, an adding machine, a desk calculator and a computer. (The syllabus can be made available if requested).

ENTRY STANDARD

Suitable candidates must possess a good principal pass in Mathematics at A Level with supporting Physics. When selecting them care is taken to note the standards of achievement in both pure and applied mathematics papers. This reduces the fear many students have of applied mathematics especially mechanics and sometime statistics.

IMPLEMENTATION

Although N.T.C. Kyambogo has been running for about 20 years the output of qualified teachers to teach Mathematics has been low considering the needs in the schools. By 1974 the output was on the average 20 teachers per year qualified to teach Mathematics. They were required to teach another subject e.g. physics, chemistry, geography and physical education. From 1975 up to now the output increased from 20 to 35. The intake every year at Kyambogo is 50 but not all qualify, others drop out and by the end of the course about 30-35 qualify. As mentioned earlier, the successful candidates are awarded the Makerere University Diploma in Education. The Ministry of Education then posts them to secondary schools. This is a drop in an ocean.

Opening up a similar college at Ngetta could alleviate the problem a little. I am saying so because many more new schools, government aided, have been opened up and they are without qualified teachers.

When the college at Ngetta was opened, I happened to be one of the few people to start the course. I started with teaching the students. Unfortunately I had to leave after two weeks to go back to my office. However I arranged a "visiting lecture" system so that every two weeks the Ministry of Education sent lecturers to teach these students. Meanwhile we were looking for suitable experienced teachers from secondary schools to be posted there on a permanent basis.

I must say that for the first half of the academic year there was no full time lecturer in Mathematics and the students depended very much on visiting lecturers from Makerere University, the Inspectorate, The National Curriculum Development Centre (NCDC) and experienced lecturers at N.T.C. (Kyambogo). By the end of the first academic year the Ministry had managed to recruit one full time lecturer.

I had to operate between my office and N.T.C. Ngetta to make sure the course did not fail. I was and still am coordinating the lecturers. I must say my ten years experience at N.T.C. Kyambogo as a lecturer, and head of mathematics department there and as senior lecturer, has enabled me to successfully organised this Double Main Course. Demanding as it is, I am glad to report that as I present this report, the first lot of qualified teachers in the double main mathematics course are doing their final exams after which they will go for a final teaching practice for six weeks.
Throughout the course the student teachers are subjected to two sessions of teaching practice, popularly known as school practice. They are seriously supervised and in the second session they receive external examiners. This year we expect about 20 qualified mathematics teachers from Ngetta. We also expect about 20 from Kyambogo and about 10 from N.T.C. Kakoba this will be a step forward.

I think our main problem is trained teachers competent enough to teach the subject effectively.

I would like to appeal to this conference and any influential bodies to help train our teachers so as to promote the standard of mathematics both nationally and internationally.

Thank you.

Due to unforseen circumstances Mr. Magino was unable to present his paper at the conference. Hence his name does not appear in the conference program.

6. COMPUTERS AND MATHEMATICS TEACHING

SOME PROBLEM SOLVING ACTIVITIES WITH A FUNCTION GRAPHING PROGRAM

R. SELDON

Introduction

A consideration of the possibility of using computers in an educational setting for concept exploration and problem solving led me to begin work on a function graphing program specifically designed for this purpose. This program has been called Investigations in Function Graphing . The first version of the program was presented to the Mathematics Education Centre conference in 1983 and has been used in introductory calculus courses for the last year. A second version of the program, which will be used in this workshop, will be introduced into classes in the near future.

Acceptance by my colleagues of the idea of using such a program in their classes has not been widespread and one of the aims of this workshop is to encourage them to do this.

The first version of the program was able to draw user defined functions on any scale, to draw first and second derivatives and integrals, to draw tangent lines and to encourage the estimation of gradients. It allowed redrawing of the graph on a different scale but, unfortunately, in a rather clumsy way. I say unfortunately because this feature of the program is probably the most important for exploratory work. Although I do not wish to emphasise the shortcomings of the original version I must admit that its slowness, together with some faults in the input structure, prevented it from being the powerful tool that I had anticipated.

In spite of its shortcomings the program has achieved some success. It certainly has provided the students with a different view of functions and has enabled them to produce graphs that would otherwise be beyond their analytic skills. They have responded enthusiastically to the use of the program, even after the initial novelty has worn off. Many

SOME PROBLEM SOLVING ACTIVITES WITH A FUNCTION GRAPHING PROGRAM

A WORKSHOP

Presented to the Mathematics Education Centre 1984 Annual Conference, Papua New Guinea University of Technology, May 1984.

Rod Selden, Papua New Guinea University of Technology

CONTENTS

Introduction

How to Operate the Program

Some Problems to Try

- Solving Equations
  - Problem 1
  - Problem 2
- Graphs on different Scales
  - Problem 3
  - Problem 4
- Graphs with Unusual Features
  - Problem 5
- Special Relationships
  - Problem 6
  - Problem 7
- Odd and Even Functions
  - Problem 8
- Calculus
  - Problem 9
  - Problem 10

students have initiated investigations of their own and have responded positively to the often surprising results that they have produced. It has been possible to design activities related to most areas of the course syllabus.

My subjective assessment at this stage is that use of the program has provided much needed variety and a valuable perspective, but that its ability to provide a significant stimulus to concept development has not yet been proved.

The exploratory power of the program is critical to its educational effectiveness. It needs to do more than just produce graphs in a somewhat magical way. The activities which will be successful will be problem solving activities in which an analytic approach to interpreting the graphs is not just possible but necessary, and for which the features of the program provide genuine tools for exploring and solving the problem. I imagine that the sort of activities which will be most successful will be those that use the computer along with the traditional tools of pencil and paper, graph paper, calculator and so on.

The second version of Investigations in Function Graphing will I hope be a more effective problem solving tool. It is faster, has a more efficient overall structure, is less error prone and has a powerful rescaling feature. I am still working on the problem of making the program psychologically sound, that is of making its use easy and comfortable so that the students can concentrate on the mathematics rather than the problems that the computer itself poses. Obviously I welcome any feedback in this area. The program is still undergoing the final stages of "debugging" and although I hope it doesn't do something crazy I also hope you will let me know if you manage to fool its foolproof system.

The activities in this workshop are mostly activities which could be used in our Foundation Year Engineering course, or in other similar courses. They illustrate the sort of activity that might facilitate concept and skill development by encouraging an investigatory and analytic approach to a problem solving situation. Please try some of them, but also think about other problems that might be approached in this way.

How to Operate the Program

The operation of the program is (meant to be) extremely simple and only a few instructions are necessary.

When you start the program you are immediately given the opportunity to enter and graph a function. Following this you will be shown a menu from which can choose amongst the various features.

After the menu is shown for the first time you can return to the menu at any stage by simply pressing the <ESCAPE> key. If the program is busy plotting a graph you may have to wait until it is finished.

After the program has drawn a graph it waits for you to press a key before it continues. Any key will do.

When you are asked to enter a number or a function you will usually be given a default choice. Simply pressing the <RETURN> key is equivalent to entering this choice. If you want to choose something else just type it over the top.

If you have trouble entering functions or using the rescaling feature help is available. Just type <?>. Functions need to be defined using the code of the basic programming language, which is fairly straightforward and follows normal algebraic conventions.

When you see the] prompt type RUN START and you will be underway!

SOME PROBLEMS TO TRY

Solving Equations

PROBLEM 1:

The rescaling feature allows you to focus on specific parts of the graph. One use of this is to get quite accurate solutions for the roots and turning points of functions. This can be related to any method currently being taught, such as the quadratic formula or Newton's method.

(a) Find the roots of the quadratic function

$$f(x) = x^2 + 2x - 7$$

and write this function in the form

$$f(x) = (x - a)(x - b).$$

Use the quadratic formula to calculate these roots and compare the two results.

(b) Complete the square on the above function and hence determine the axis of symmetry and the minimum point of the graph. Inspect the graph and see if it agrees with your calculations.

Problem 2:

(a) By drawing their graphs on the same diagram find the points of intersection of the two functions...

$$f(x) = x^3$$

and $g(x) = 2x^2 + 5x - 6$

(b) Show that the x coordinates of these points of intersection are also the roots of the function

$$h(x) = x^3 - 2x^2 - 5x + 6$$

Why is this?

(c) Find the points of intersection of

$$m(x) = x^2 - 2x - 5$$

and $n(x) = \frac{-6}{x}$

Are the x coordinates of these points the same as for f(x) and g(x)? Explain this.

Graphs on Different Scales

Problem 3:

(a) Draw the graph of

$$f(x) = (x - 1)(x + 2)(x - 3)$$

Observe what happens as you increase the scale. Why is it necessary to make different adjustments in the horizontal and vertical directions?

(b) Continue to increase the scale until the x-axis goes from -100 to 100. What is the appropriate range of values for the y axis? Is this what you would expect?

(c) On the same graph draw the function

$$g(x) = x^3 .$$

What happens? Can you explain this?

(d) Use the rescale window to focus back in on the origin. For what values of x is the difference between the two graphs observable?

Problem 4:

(a) Consider the two functions

$$f(x) = 10x^2$$

and $$g(x) = e^{0.1x}$$

Which of these functions will have larger values when...

(i) x is negative

(ii) x is zero

(iii) x is small and positive

(iv) x is large and positive.

(b) Draw these two functions on the same graph. Are your predictions of part (a) observable? Is it true that exponentials dominate quadratics when x is large? Does it look like this on the graph?

(c) Increase the scale in the x direction until the x-axis goes up to about 200. What happens? How does the graph compare with the same graph on a smaller scale?

(d) What do you think the graph of

$$h(x) = \frac{e^{0.1x}}{10x^2}$$

will look like when drawn on a small scale $(-10 < x < 10)$ and when drawn on a large scale $(-200 < x < 200)$. Draw a sketch on paper and then use the program to check your predictions.

Graphs with Unusual Features

Problem 5:

(a) Draw the graph of

$$f(x) = (x + 6)^3 + \frac{1}{x}$$

with the x-axis going from -10 to 10. $f(x)$ has two terms, namely $(x + 6)$ and $1/x$. Which of these does the graph look most like?

(b) Considering that $f(x)$ contains the term $1/x$ what happens to $f(x)$ as x approaches zero? Is the function defined when $x = 0$? Can you see all this on the graph?

(c) To observe the behaviour of the graph as x approaches zero use the rescaling window to focus on the section of the graph near the y-axis.

Special Relationships

Problem 6:

(a) Draw the graphs of

$$f(x) = \sin(x)$$

and $g(x) = x$

over the domain $-3 < x < 3$. For what values of x do sin(x) and x have similar values?

(b) What do you think a graph of

$$h(x) = \frac{\sin(x)}{x}$$

would look like? Draw a sketch on paper and then use the program to check your prediction.

(c) What can you say about

$$\lim_{x-0} \frac{\sin(x)}{x} \quad ?$$

Problem 7:

(a) It is easy to show that if

$$f(x) = e^x$$

then $$f'(x) = e^x \lim_{h-0} \frac{(e^h - 1)}{h}$$

To examine the $\lim_{h-0} \frac{(e^h - 1)}{h}$

we can examine the graphs of

$$g(x) = e^x - 1$$

and $$h(x) = x.$$

Sketch these graphs either on paper or using the program.

(b) What do you think the function

$$m(x) = \frac{(e^x - 1)}{x}$$

would be like? Draw a sketch on paper and then use the program to check your prediction.

(c) What can you say about

$$\lim_{h-0} \frac{(e^h - 1)}{h} \quad ?$$

Does this tell you what the derivative of e^x is?

Odd and Even Functions

Problem 8:

For each of the following functions you should...

(a) Decide whether the function is odd or even.

(b) Consider what happens to the function when

(i) x is large and negative

(ii) x is close to zero

(iii) x is large and positive.

(c) Use this information to draw a sketch of the function on paper.

(d) Check your prediction by drawing the graph on the computer.

The functions are

(i) e^{x^2}	(ii) e^{-x^2}
(iii) xe^{-x^2}	(iv) $x\cos(x)$
(v) $\frac{\cos(x)}{x}$	(vi) $\frac{1}{1 + x^2}$
(vii) $\frac{x}{1 + x^2}$	(viii) $\frac{x^2}{1 + x^2}$

Problem 9:

For this activity almost any function will be suitable. A simple cubic makes a nice example to start with but then try other functions, such as $\cos(x^2)$, which students do not normally look at in this context.

(a) Draw the graph of

$$f(x) = (x - 3)(x + 1)(x + 3)$$

over the domain $-4 < x < 4$.

(b) Inspect this graph closely and decide...

(i) for what values of x the slope of the function is zero

(ii) for what values of x the slope is negative, and for what values of x it is positive.

(iii) what the largest (maximum or minimum) slope is between each of the turning points.

(c) Use this information to sketch on paper a graph of the gradient function. Then use the computer to check your prediction.

(d) Repeat the above activity on the gradient function itself. The result should be a sketch of the second derivative. Check your prediction by using the computer to draw the second derivative.

Problem 10:

(a) Draw the graph of

$$f(x) = (x - 3)(x + 1)(x + 3)$$

(b) Inspect this graph closely and decide...

(i) for what values of x an integral will have a turning point

(ii) for what values of x an integral will have a positive slope, and for what values of x it will have a negative slope.

(iii) for what values of x an integral will have a point of inflection

(iv) by estimating the areas under the graph try to determine the differences between the y values of the turning points of an integral.

(c) Use this information to sketch on paper the rough shape of an integral. Use the program to draw an integral and check your prediction.

(d) Try drawing a second integral through a different initial point. Does this integral have the same shape as the first integral? Is the difference between the two integrals a constant as you would expect?

MICROCOMPUTERS AND EDUCATION IN FIJI

FRED S DEWA

The University of the South Pacific
School of Education
Suva, Fiji.

ABSTRACT : *January 1984 saw the introduction of microcomputers in the Secondary Schools in Fiji. This paper traces the short history of microcomputers in Fiji Schools and discusses current research being undertaken at USP.*

A very short history

When the Fiji Government announced its intentions to introduce microcomputers in Fiji schools at the end of 1983, the International School in Suva, Lelean Memorial School, and Suva Grammar School were the only schools in Fiji with any computers. The government at the same time reduced the import duty on microcomputers to approximately 5%. In January 1984 the Ministry of Education mounted the first of its computer training courses for selected teachers. The course lasted two weeks and by the end of the first school term of 1984 the government had equipped 11 secondary schools with 6 microcomputers each. The government also indicated that it hopes to equip all secondary schools in Fiji with microcomputers within three to five years.

The government's favourable attitude to computer education aroused such an interest, computer classes sprang up all over Suva. These classes, run by business houses, advertised topics from introductory programming and keyboard operation to word processing and advanced programming. USP currently runs two non credit courses in introductory programming through the mathematic department and the extension services.

Hardware available over the counter in Suva range from the 1K machine selling at $F90.00 to the 64K machines selling at around $F500.00.

RESEARCH AT USP

Introduction

USP purchased it's first microcomputers in 1980 for various purposes. In the mathematics department the main use has been for teaching computer programming. Some third year students have used the machines for solving problems in numerical analysis. The first real attempt at using microcomputers to teach mathematics at USP is by the author at the beginning of the 1984 academic year. This came about firstly because of the author's interest and secondly as part requirement of a Masters programme.

The Project Objectives

The project is aimed at investigating the use of micros for teaching mathematics to Foundation Students at USP.

Foundation Students

These are pre-degree students who have attained New Zealand University Entrance standard. The current population of Foundation Students have not had any computer training at all. A vast majority of the students are on government scholarship which means they are influenced by their governments on their choice of courses. As a result a student may be taking Foundation maths at the request of his sponsor.

Foundation Mathematics

This is a one year programme. Topics taught include algebraic processes, functions and their graphs, series, trigonometry, elementary differentiation and integration with further topics selected from complex numbers, logic, elementary linear algebra, analytic geometry, vectors, probability, statistics and computing. Four topics are selected for each of the two semesters of the Foundation year.

Foundation mathematics are taught in the conventional high school method i.e. a teacher with a class of say thirty students. The students are not subjected to lectures. Class sizes vary between 20 and just under 40. Fifty minute classes are timetabled five times a week. Each topic is allocated roughly 3 to 4 weeks and a fifty minute test is administered at the end of each topic. The four tests taken during the semester is worth 50% of the final semester grade, while the other 50% comes from the final three hour examination at the end of the semester which examines the four topics taught during the semester.

Outline of the Project Method

The author teaches two Foundation groups. Group A are students wishing to major in Industrial Arts and Group B are Social Science Students. The topics to be taught in semester one of 1984 are algebra, functions and their graphs, trigonometry and complex numbers. Up to the time of writing this, May 1984, algebra, functions and their graphs and trigonometry have been covered in that order.

Algebra was taught to the two groups A and B in the conventional classroom/teacher situation and a test administered. For graphs and their functions the two groups were split into two further groups A1, A2 and B1, B2. Groups A1 and B1 were taught with the aid of microcomputers while groups A2 and B2 were taught in the conventional way then tested. The intention was to switch the machine to groups A2 and B2 when trigonometry was taught. Administrative problems prevented this, so the switch will have to wait for another topic.

Considerations

Administration, hardware, and software are among problems to be considered. The ideal situation is a fully equipped computing laboratory. The project at USP used the author's office equipped with 3 microcomputers with a portable disk drive. During the three weeks that the students were taught with the use of the microcomputers, the machines were available to the students from eight in the morning to four in the afternoon. The first couple of lessons were used to instruct the students how to operate the machines. The idea was to have a booking system such that only two students are on a machine at any one time. This idea was abandoned when it was noted that students were quite happy to sit around the machine in larger groups with one operating and the rest making suggestions about inputs and debating the reasons for the different outputs. The verbal exchanges were quite interesting. Students who were not supposed to use the machines had to be reminded on this on various occasions.

Hardware is restricted to what is available and what can be afforded. Graphics capability and ease of operation are also important when considering hardware. The project at USP uses the Atari 800XL, which has 9 graphic modes the highest resolution being 320X160 and various colours to choose from. A disk drive, Atari 1050, is employed instead of a tape drive for speed of loading programs since the students often lose the program from memory. Some domestic TV sets come in dual mode, TV/Monitor, and they give a sharper picture than the single mode TV.

Software is perhaps the most important part of the project. Software can be purchased or home made. The programs for the project are written by the author and run by the students. This method is used due to the inability of students to write programs. When students are in a position to write programs it would be more beneficial to ask students to write programs to demonstrate mathematical concepts. Some mathematical topics are more suitable to computer treatment than *others. The USP project started by teaching algebra which is not so suitable for computers, but the second topic, functions and their graphs is ideal for computers.*

Conclusions

The results of the first two tests on algebra and functions/graphs are still being analysed.

A by product of the project is the production of a video tape of the program being run, with accompanying commentary. This is seen as a possible media for the extension services.

VENUE - Audio Visual Theatre, University Library.

FORMAT - Each presentation will be of half an hour. A paper will be talked to for approximately. twenty minutes leaving ten minutes for questions and discussion.

PROGRAM

Thursday - 24th May

8:05 - 8:15	Introduction Philip Clarkson - M.E.C., PNGUT
8:15 - 8:30	'Welcome' Vice-Chancellor - Mr. Moromoro
	Points to Consider in Teaching Mathematics Chairperson - Philip Clarkson
8:30 - 9:00	'Is there a place for this kind of thing...?' Basil Landau - Maths Dept., PNGUT
9:00 - 9:30	'Representations on paper of 3-D geometrical shapes' Prof. Claude Gaulin - Education Faculty, Laval University, Canada.
9.30 - 10:00	'Formal operational thought with PNGUT students' Glen Lean - Maths Dept., PNGUT
10:00 - 10:15	MORNING TEA
	Post Year 10 Mathematics Chairperson - Kay Owens
10:15 - 10:45	'The quality of the grade 10 school leavers pool' Lawrence Ros - Maths Dept., Goroka Teachers College
10:45 - 11:15	'Distance versus face-to-face learning: a mathematics test case' G.S. Kaeley - Extension Studies Dept., UPNG.
11:15 - 11:45	'Minimal remediation: Pre-test as a kick in the pants' John Gough - Education Dept., UPNG.
11:45 - 1:00	LUNCH
	PNGUT Year 12 Entry Chairperson - Helen Whippy
1:00 - 1:30	'Are Grade 12 University Entry Students Deficient in Their Mathematics Education?' Prof. Kathleen Collard - Maths Dept., PNGUT.

	'Microcomputers and Education'
1:30 - 2:00	'Microcomputers as mathematics teachers' Prof. Don Joyce - Maths Dept., UPNG.
2:00 - 2:30	'Microcomputers in education in Fiji' Fereti Dewa - School of Education, U.S.P.
2:30 - 3:00	AFTERNOON TEA
3:00 - 4:00	'Some problem solving with a function graphing program - a workshop' Rod Seldon - Maths Dept., PNGUT (This session will be held in the microcomputer laboratory on the ground floor of the Accountancy and Business Studies building).
Friday 25th May	
	'Cultural Aspects of Mathematics Education'
	Chairperson - Neil Roberts
8:15 - 8:45	'Basic computation skills progress in grade 10, Consumer Mathematics' Dale Nelson - Education Department, Yap, F.S.M.
8:45 - 9:15	'Mathematical abilities at the upper secondary school in Fiji' Leonard Raj - School of Education, U.S.P.
9:15 - 9:45	'What is it like being a mathematics teacher in a secondary school in Papua New Guinea' Naomi Wilkins - M.E.C., PNGUT
9:45 - 10:15	MORNING TEA
	'Reports on Research'
	Chairperson - Chris Wilkins
10:15 - 10:45	'Word Problems at Year 6 Level' Philip Clarkson,- M.E.C., PNGUT.
10:45 - 11:15	'First and last, one and two, letter and word: Concept Formation in PNG Community Schools' Dennis Moore - Education Research Unit, UPNG.
11:15 - 11:45	'The prediction of simple loci' Glen Lean - Maths Dept., PNGUT
11:45 - 1.00	LUNCH

'University Mathematics Teaching (1)

Chairperson - Glen Lean

1:00 - 1:30 'Review of Mathematics Education in Papua New Guinea'
Robert Maioni - Ministry of Education

1:30 - 2:00 'Some current mathematical aspects in Surveying Education'
Ass. Prof. Peter Done - Surveying Dept., PNGUT.

2:00 - 2:30 'An experimental approach to teaching mathematics'
Prof. Don Joyce - Maths Dept., UPNG.

2:30 - 3:00 AFTERNOON TEA

University Mathematics Teaching (2)

Chairperson - Philip Clarkson

3:00 - 3:30 'What problems can tertiary students solve?'
Chris Wilkins - Maths Dept., PNGUT.

3.30 - 4:00 'A mathematics profile of grade 10 leavers at PNGUT, 1984'
Helen Whippy - Maths Dept., PNGUT.

The following M.E.C. Reports have been published:

No. 1: A. EDWARD 'Report for 1978, including a Short History of the Mathematics Learning Project and Mathematics Education Centre up to 1977', 1978.

No. 2: A.J. BISHOP 'Spatial Abilities in a Papua New Guinea Context', 1978. (out of print)

No. 3: C. MEEK and N. FERIL 'A Pilot Study into the Level of Comprehension of Some Words taken from the Papua New Guinea Community Schools Mathematics Syllabus', 1978.

No. 4: A.J. BISHOP 'Developing Spatial Abilities', 1978. (out of print)

No. 5: A. EDWARDS 'A Study of the Issues Involved in Establishing the Mathematics Education Centre and a Report on its Work from December 1978 to May 1979', 1979. (out of print)

No. 6: A. EDWARDS and A. BAJPAI 'Preliminary Report on the Basic Arithmetic Test carried out in Post-Secondary Institutions in Papua New Guinea in 1979', 1979 (out of print).

No. 7: A. EDWARDS 'Final Report on the Establishment of the Mathematics Education Centre', 1979 (out of print).

No. 8: G.A. LEAN 'The Training of Spatial Abilities: A Bibliograph', 1980.

No. 9: G.A. LEAN 'An Investigation of Spatial Ability Among Papua New Guinea Students', 1980. (out of print)

No. 10: G.A. LEAN and M.A. CLEMENTS 'Spatial Ability, Visual Imagery and Mathematical Performance', 1980. (out of print)

No. 11: P.L. JONES 'The Introductory Mathematics Programme at the Papua New Guinea University of Technology – A Survey of Student Opinion', 1980. (out of print)

No. 12: P.L. JONES 'A Study of Proportional Reasoning', 1980. (out of print)

No. 13: M.A. CLEMENTS and G.A. LEAN 'Influences on Mathematical Learning in Papua New Guinea: Some Cross – Cultural Perspectives', 1981, (2nd edition). (out of print)

No. 14: M.A. CLEMENTS 'Careless Errors Made by Sixth-Grade Children on Written Mathematical Tasks', 1981 (2nd edition).

No. 15: P. SULLIVAN 'The Standards Debate: An Alternate Strategy', 1981. (out of print)

No. 16: P.L. JONES 'Mathematical Language and the Learning of Mathematics in Papua New Guinea', 1981. (out of print)

No. 17: F. AUSBURN and L. AUSBURN 'Perception, Imagery and Education in Developing Countries', 1981. (out of print)

No. 18: M.A. CLEMENTS and P.L. JONES 'The Education of Atawe', 1981. (out of print)

No. 19: P.L. JONES 'Learning Mathematics in a Second Language: A Problem with More and Less', 1981.

No. 20: M. SIFORD 'A Study of Algebra in the Provincial High Schools Curriculum', 1981.

No. 21: F. AUSBURN and L. AUSBURN 'Cognitive Styles and Visual Skills in Papua New Guinea Students', 1982. (out of print)

No. 22: P.C. CLARKSON 'Mathematics Learning in PNG National High Schools: The Analytic – Visual Variable', 1982. (out of print)

No. 23: P.C. CLARKSON 'Mathematics Learning in PNG National High Schools: A Comparison of Years 11 and 12 Across Schools', 1982. (out of print)

No. 24: P. SULLIVAN 'Practical Aspects of Drug Calculations', 1982.

No. 25: P.C. CLARKSON 'Year 12 Students' Mathematical Performance on the 1980 and 1981 External Examinations', 1983. (out of print)

No. 26: P.C. CLARKSON 'Types of Errors Made by Papua New Guinean Students', 1983.

No. 27: P.C. CLARKSON 'Papua New Guinean Students' Causal Attributions for Success and Failure in Mathematics', 1983.

No. 28: P. SULLIVAN 'Remediation of Basic Arithmetic Weaknesses in Post-Secondary Students', 1983. (out of print)

No. 29 P. SULLIVAN 'Writing Educational Materials: A Report of a Staff Training Program in Papua New Guinea', 1983.

No. 30 P.C. CLARKSON 'Papua New Guinea Students' Perceptions of Mathematics Lecturers', 1983.

No. 31 P.C. CLARKSON 'Error Analysis and Attribution Theory', 1983.

No. 32: P. SULLIVAN 'An Investigation of the Mathematics Education of Nurses in Papua New Guinea', 1984.

No. 33: P.C. CLARKSON and G. LEDER 'Causal Attributions for Success and Failure in Mathematics: A Cross-Cultural Perspective', 1984.

The following M.E.C. Technical Reports have been published:

No. 1: P. SULLIVAN 'Report of the First Conference to Prepare Mathematics Material for Post-Secondary Courses', 1981. (out of print)

No. 2: P.C. CLARKSON 'Report of the 1980 National High School Year 12 Mathematics Marking Panel', 1981.

No. 3: 'Annual Report, 1980', 1981.

No. 4: 'Annual Report, 1981', 1982. (out of print)

No. 5: P.C. CLARKSON 'Report on the 1981 National High School Year 12 Mathematics Examinations', 1982. (out of print)

No. 6: P.C. CLARKSON 'Pre-University Mathematics Course', 1982.

No. 7: P. SULLIVAN 'Report of Second Program to Prepare Mathematics Materials for Post-Secondary Courses', 1982. (out of print)

No. 8: P.C. CLARKSON 'Report on the 1982 National High School Year 12 Mathematics Examinations', 1983.

No. 9: 'Annual Report, 1982', 1983.

No. 10: 'Problem Solving through the Curriculum', 1983.

No. 11: P. SULLIVAN 'Report of the Third Program to Prepare Mathematics Materials for Post-Secondary Courses', 1983.

No. 12: P.C. CLARKSON 'Report on the 1983 National High School Year 12 Mathematics Examination', 1984.

No. 13: 'Annual Report, 1983', 1984.

No. 14: A. KOIGIRI 'Year 12 Students' Mathematical Performance on the 1982 External Examination', 1984.

No. 15: W. KALEVA 'Year 12 Students' Mathematical Performance on the External Examination, 1983', 1984.

No. 16: B. SOUTHWELL 'Problem Solving With Calculators for Grade 10 Students', 1984.

BEST COPY AVAILABLE

Other publications include:

P. SULLIVAN 'An Introduction to Nursing Mathematics', 1981. (out of print)

P.C. CLARKSON (ed) 'Research in Mathematics Education in Papua New Guinea, 1981', Proceeding of the First National Conference on Mathematics Education, May, 1981. (out of print)

P.C. CLARKSON and P. SULLIVAN 'Vocational Mathematics', 1982.

P.C. CLARKSON (ed) 'Research in Mathematics Education in Papua New Guinea, 1982', Proceedings of the Second National Conference on Mathematics Education, May, 1982.

P.C. CLARKSON (ed) 'Research in Mathematics Education in Papua New Guinea, 1983', Proceedings of the Third National Conference on Mathematics Education, May, 1983.

P.C. CLARKSON (ed) 'Research in Mathematics Education in Papua New Guinea, 1984', Proceedings of the Third National Conference on Mathematics Education, May, 1984.

PRIME - 'The Journal for Pacific Regional Information on Mathematical Education' is also published by the Mathematics Education Centre.

*Those reports indicated as 'out of print' should be available from the Matheson Library Papua New Guinea University of Technology on inter library loan.